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CARRIER OPTIMIZATION LAUNCH ALGORITHM: AN OPTIMIZATION MODEL TO MAXIMIZE THE NUMBER OF TACTICALLY TASKED SORTIES UNDER CONSTRAINT RESTRICTION

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Timely arrival of a Carrier Battle Group (CVBG) at an assigned location is extremely important for successful execution of tactical, strategic or logistic operations. Weather conditions, tasking requirements, battle group defense and transit restrictions all impact on the CVBG's rate of advance while in transit to its newly assigned station. If these variables are not carefully considered and accounted for, then the CVBG, in all likelihood, will arrive late at its destination. This paper develops a mathematical model, which incorporates these transit variables, to assist the Battle Group Commander in successfully planning flight operations which will meet both CVBG defensive and routine tasking requirements and as well as arrive at its destination on time. This model through the use of elastic constraints will provide the commander with maximum flexibility and allow him to determine any inconsistencies between desired tasking and transit requirements.

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I. BACKGROUND

As a result of increased terrorist activity and the volatile nature of many third world countries, greater emphasis has been placed on enhancing the mobility and responsiveness of our naval and military forces. Our ability to quickly respond with a massive show of force or conduct effective air, sea or land attacks on short notice are key elements in maintaining our military superiority.

One of the most mobile and powerful forces available in response to the need for protecting United States interests and citizens abroad is the Aircraft Carrier Battle Group (CVBG). The CVBG, which usually consists of an aircraft carrier, several combat warships of various types (CG, DD, DDG, FF, FFG) and other support vessels (AOE), has the advantage of having its own air force available 100% of the time. When a CVBG is prepositioned (forward-deployed), it is possible to project an airborne military force necessary to respond within hours or at most a few days to most crises. Furthermore, a CVBG combined with an amphibious assault group results in a combat force capable of handling operations involving air, sea and land forces.

If a CVBG is not forward deployed then it must generally conduct an extensive transit prior to being within strike range of a selected target. Consequently a conflict now arises between the need for self-protection and the requirement to arrive within strike range at the specified time. During transit the CVBG must consider how best to defend itself against air, surface and subsurface threats which it may encounter enroute to the target area. Since use of CVBG air assets for self-defense purposes will encumber the battle group's scheduled

arrival onstation, the CVBG commander must carefully analyze the interaction between the defensive requirements and the transit requirements.

The interaction between the defensive requirements and the transit time constraints is directly related to the depth of air defense desired (number of aircraft needed), time of aircraft launch and recovery (day or night; type of launch and recovery), available aircraft (maximum number which can be launched), **Position of Intended Movement** (PIM: where and when), and weather conditions (wind, visibility, cloud ceiling, sea state). Of these interactions, weather is the most critical factor affecting the battle group progress along its PIM while simultaneously conducting flight operations.

Specifically, wind direction and speed may require the CV to turn in order to obtain the required wind conditions across the deck for a successful launch and recovery of aircraft. If the origin of the wind is from a direction which is the reciprocal of the CVBG's PIM, then the only influence is a change in the CV's speed. But, if the wind originates in any other direction (which is usually the case), then the CV will be required to deviate from PIM thereby increasing the actual distance that must be traveled in order to arrive at the assigned destination. The length of deviation from PIM will be dictated by the time required to launch and recover aircraft during a given flight cycle. Total flight cycle launch and recovery time is affected by the number of aircraft to be launched for the present cycle, the number of aircraft to recover from previous cycles, the time of the day, and weather considerations such as sea state and visibility. The minimum number of aircraft to launch is a function of the defensive posture desired. Time of day is significant in that night launches and recoveries usually require more

time than daytime operations. An increase in sea state or reduction in visibility will also increase launch and recovery times.

The objective of the battle group commander is to find the appropriate level of air operations which will meet the CVBG defensive posture while allowing the CVBG to arrive as scheduled. Presently all computations for determining an optimal mix level of air operations are accomplished manually. Although quite simple, these computations can be very cumbersome. To obtain a manual solution the CVBG commander first determines the defensive requirements with respect to aircraft launches; then, based upon the weather for each cycle, the CVBG dead reckoned track is manually computed for the flight operations period. The result is an **Estimated Position** (EP) of the CVBG at the end of flight operations which may be behind, ahead or on the PIM. If the EP is significantly behind the intended position, it is deemed infeasible and the battle group commander must modify either the desired defensive capability or the proposed mission time constraints in an attempt to obtain a feasible solution. If the EP is significantly ahead of the PIM, the commander can either accept the resulting schedule as feasible, but not optimal, or determine that the solution is infeasible and attempt to obtain an optimal solution. If the estimated position is exactly on PIM, then the optimal feasible solution has been obtained. It should be obvious at this point that much time may be wasted in attempting to obtain a feasible solution, which may not necessarily be the optimal solution. As long as more sorties can be launched and the time constraint not violated, then there is a better solution which should increase defensive posture. Thus, there must exist a combination of aircraft sorties of different lengths (single cycle, double cycle, etc.), based upon requirements (posture, weather time, PIM), that can be launched

and recovered which would maximize the number of sorties and still allow the CVBG to meet its transit constraint. This desire to have the maximum number of sorties while meeting certain constraints leads to employing a mathematical modeling approach for the battle group commander to solve his dilemma.

Specifically, a mathematical model using **integer linear programming** can be designed which will provide the battle group commander with a close approximation to the optimal solution while meeting transit constraint and tasking requirements. An approximation to the model is provided instead of the exact optimal solution because of the necessity to use a desktop computer and the drawbacks provided in the next paragraph. Maximization of the number of sorties flown during flight operations is subject to the following constraints:

- A minimum defensive posture is obtained.
- A minimum distance along PIM is traveled.
- A maximum distance along PIM is not exceeded.
- Sorties do not exceed the number of aircraft:
  - available for each cycle,
  - available overall.
- A minimum number of flight hours is flown by each type of aircraft.
- The time to launch, recover and re-spot aircraft does not exceed the cycle time allotted.
- Nonnegative sorties.

Three major drawbacks of the model are: (1) it produces results which are only as accurate as the data that is input (weather, ship movement); (2) it does not include any operations that may deviate from the pre-planned flight data (emergencies); (3) and in order to maintain linearity in the model, initial approximations have to be made as to the expected number of degrees and the

associated time to turn into the wind for each cycle, causing position data and sortie numbers not to be exact.

The actual model is logically divided into three phases. The first phase will involve the input of parameters required to compute actual numbers for the constraints in the linear programming problem. Inputs such as PIM, weather, minimum required sorties, flight operations/cycle scheduling and available aircraft will provide the coefficients for the decision variables in both the objective function and the constraint functions. Phase 1 is accomplished using a data input/flight analyzer program known as the **Carrier Optimization Launch Program** (COLP). The second phase is execution of the model using the data input during phase 1.

Phase 3 restructures the output from the previous phase and produces a recommended flight plan for flight operations along the estimated track for the CVBG. The generated flight plan provides a schedule of sorties providing the greatest number of flights (best defensive posture) while allowing the CVBG to be on PIM for the start of the next flight operations period or at its assigned station location on time. The estimated track which the model generates also provides an EP for the CV at the end of each flight cycle as well as an EP for the start of the next flight operations period or assigned rendezvous time (Figure 1).

If an optimal feasible solution cannot be found to the problem, then the battle group commander should attempt one or more of the following if possible:

- reduce the number of required sorties (decrease defense posture);
- decrease the length of the flight operations period;
- shorten the distance to the onstation location;
- revise/adjust PIM;
- increase CV sprint velocity between launch cycles;

- increase sprint velocity when not in flight operations;
- consider remaining on PIM heading with an increased velocity if wind originates from behind the carrier and is light.

Changes to a single parameter or a combination of the parameters listed above may result in an optimal or at least acceptable solution to the problem.

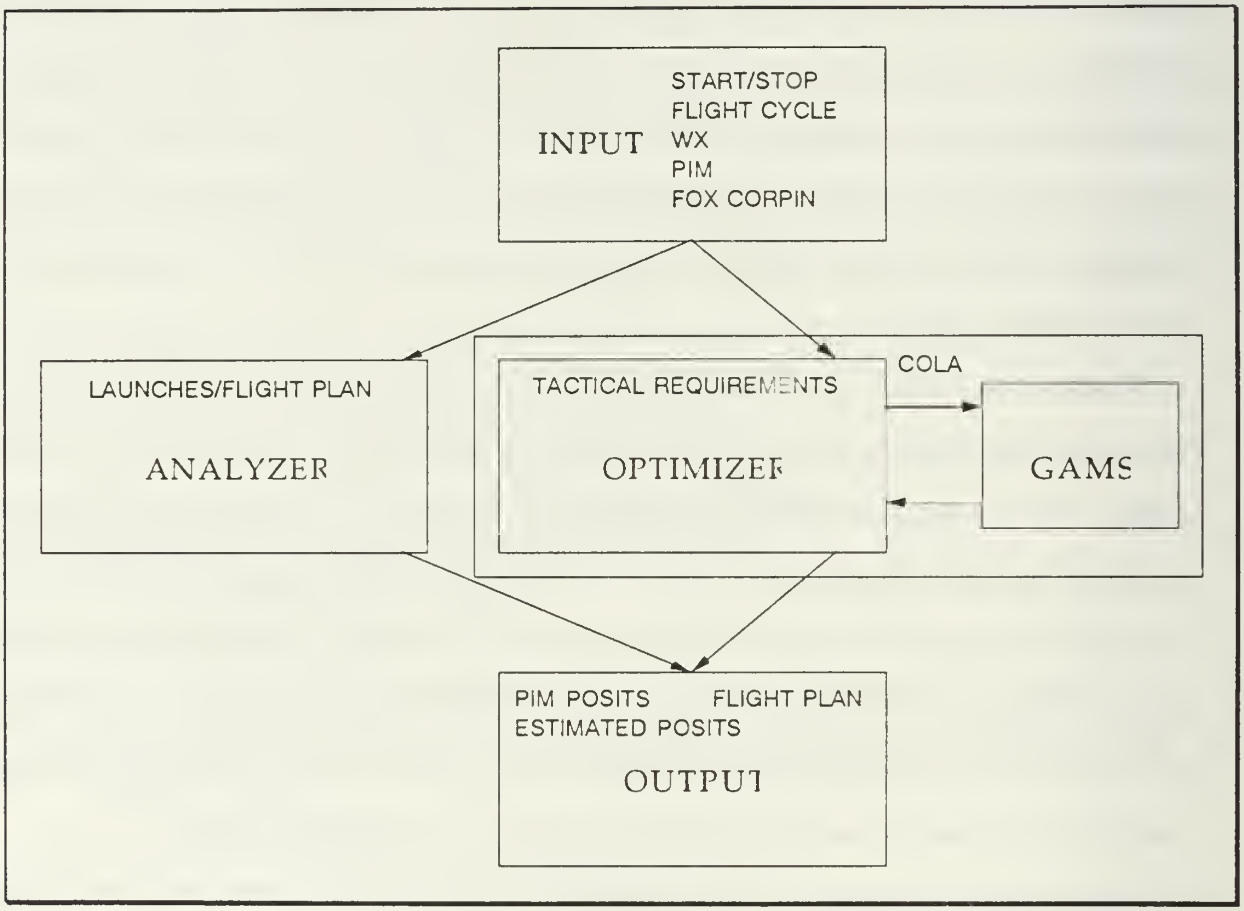


Figure 1. Carrier Optimization Launch Program (COLP)

If, after adjusting combinations of various parameters, an optimal or near-optimal solution cannot be attained then the entire tactical plan must be re-evaluated. The results of this model provide the battle group commander, or other user, with a means of evaluating the CVBG tactical plans. The model looks at the overall

combination of time constraints and defensive requirements, and by providing an optimal or near-optimal flight plan substantially reduces the probability of canceling flight operations, as is frequently the case, in order to arrive onstation as directed. As a tactical decision aid it provides the user with an approximation to the maximum number of aircraft that can be launched. In reality, a scheduler will generally not attempt to fly the determined maximum number of sorties, but will, instead, schedule the daily flight operations to be within the maximum limits allowing successful PIM transit.

II. THE MODEL

The purpose of the model is to determine the appropriate mix of tactical aircraft, of differing flight durations, to launch during each cycle, in order to maximize the number of tactical aircraft sorties conducted (during a given flight period) in support of carrier battle group defense while simultaneously allowing the CVBG to meet its transit requirements. The stated objective along with associated restrictions fall into a category of well defined problems which are usually solved by an optimization model using either linear or nonlinear programming. The objective, in this particular problem, is to maximize the number of sorties subject to the following constraints: available aircraft, transit requirements, tactical and logistic requirements, available time and flight hour requirements, and carrier air group restrictions.

The optimization model used by the **Carrier Optimization Launch Algorithm** (COLA) is a relaxed mixed integer linear programming model. It uses problem variables and constraints which are prepared by the COLP and formatted for solution by the **General Algebraic Modeling System** (GAMS). A relaxed mixed integer linear programming model was selected for two reasons. Linear programming was chosen over nonlinear programming since the objective function and all but two of the constraints are naturally linear. The two constraints which are, in reality nonlinear can be approximated very closely with linear functions. Integer programming is the preferred method of linear programming to use since the desired solution is the number of sorties (an integer quantity) to fly during a flight operations period. Since COLA is designed for operational use by either ship's company or embarked staff, mainframe computer

systems will not be available; therefore the generation time for a proposed flight plan by a typical computer used aboard ship must be of a tolerable duration. Since COLA is primarily intended to be used as a decision aid, (i.e. an exact solution is not required), a tolerable solution can be achieved by using a relaxed mixed integer programming method.

Furthermore, since approximate solutions are permissible, violation of the original constraints may be allowed by using elastic variables to soften the constraint equations. An elastic variable is a nonnegative variable which is added to all "greater than" constraints and subtracted from all "less than" constraints and has a high associated penalty coefficient in the objective function. Although the use of an elastic constraint will reduce the optimal value of the objective function, it will allow the model to have a feasible solution.

There are two constraints to which elastic variables are not permissible, specifically the total aircraft available and cycle period length. This is necessary since aircraft assignment to sorties can only be accomplished with the aircraft actually on board the carrier and use of an elastic variable would imply that an unlimited number of aircraft are available. Furthermore, there must be strict compliance with the cycle period length during fixed cycle operations.

The model to *maximize sorties* is subject to:

- Minimum required aircraft airborne
- Maximum sorties of aircraft type allowed
- Maximum sorties per cycle
- Maximum aircraft available
- Minimum flight hours per aircraft
- Cycle time limits
- Minimum distance traveled along PIM
- Maximum distance traveled along PIM

- Maximum acceptable delay per cycle (optional)

A. VARIABLES

1. Decision Variables

$x_{i,j,k}$ The number of aircraft of type j to launch at the start of cycle i for a length of k cycles.

$i =$ 1, 2, ..., scheduled cycles + 1.

$j =$	1	F-14
	2	F/A-1i
	3	A-6
	4	EA-6
	5	E-2
	6	S-3
	7	ES-3
	8	KA-6 (TKR)
	9	COD

Note 1: Launch and recovery times for all aircraft types are considered; however, the model does not attempt to maximize sorties of the tanker or COD since they are primarily transient requirements.

Note 2: Helicopters were not included in this model since they do not influence overall launch and recovery times during a cycle.

Note 3: The number of decision variables could have been reduced by a factor of 7 if the type of aircraft were not considered. This would not allow the model to determine if the distance traveled along PIM could be increased by the selection of a longer sortie vice multiple shorter sorties to meet the aircraft requirements.

$k =$	−1	nonoriginating recovery
	0	yo-yo launch (launches and recovers in same cycle)
	1	single cycle sortie
	2	double cycle sortie
	3	triple cycle sortie
	4	nonreturning launch

If $k = -1$, 0, or 4 the sortie will be considered a scheduled transient sortie and will not be considered an option for the decision variable. Otherwise the sortie is to be considered part of the decision process.

Since some aircraft types sortie lengths and launch cycle combinations are not practical, the decision variable for that combination automatically defaults to 0.

2. Soft constraint variables

These are variables which are incorporated into each of the constraints to allow the user to soften the effect of the constraints. In the objective function they will have a coefficient with a substantial negative value which will prevent their use if they are not necessary.

$S1_{i,j}$ – variable associated with softening the minimum airborne aircraft constraint.

$S2_{i,j}$ – variable associated with softening the maximum number of aircraft of type j allowed to fly in cycle i.

$S3_i$ – variable associated with softening the maximum number of sorties per cycle.

$S4_j$ – variable associated with softening the minimum flight hour requirement.

$S5$ – variable associated with softening the maximum distance required to travel on the PIM.

$S6$ – variable associated with softening the minimum distance required to travel on the PIM.

$S7_i$ – variable associated with softening the optional cycle delay constraint.

3. Coefficients

$FR_{i,j,k}$ – The minimum number of aircraft type j required to meet a scheduled tactical or logistic requirement launching at cycle i for a length of k cycles.

$OR_{i,j}$ – The number of aircraft type j required airborne during cycle i to meet generic or other tasking requirements.

$AR_{i,j}$ – The number of aircraft type j required airborne during cycle i. This airborne requirement meets specified fixed requirements plus additional generic tasking.

$$AR_{i,j} = \sum_{k=1}^{3} FR_{i,j,k} + \sum_{k=2}^{3} FR_{i-1,j,k} + FR_{i-2,j,3} + OR_{i,j}$$

$AC_{i,j}$ – The number of aircraft type j available for each cycle i. This will incorporate an exponential decrease in the number of aircraft available at cycle 1 to account for maintenance downing effects.

LC_i – The length of cycle i.

LT_i – The average time to launch an aircraft in cycle i. LT_i is a function of sea state, wind, ceiling and visibility.

RT_i – The average time to recover an aircraft in cycle i. RT_i is a function of sea state, wind, ceiling and accounted for in this average time.

LG_i – The lag time between launch and recovery of aircraft in cycle i. This could be positive if there is a delay between the last launch and first recovery or negative to signify an overlap in the two operations.

$$LG_i = K_0 + K_1(\text{ number of launches}_i)$$

RS_i – The average respot/rearm time required per aircraft for each cycle.

TI_i – The time to turn into the wind for cycle i.

TO_i – The time to turn back to PIM after the launch and recovery of aircraft in cycle i.

DELTA – The time between the end of the present flight operations period and the start of the next flight operations period or elapsed time to arrive at the next rendezvous point.

A – The maximum distance allowed to be ahead of PIM (for entire transit).

B – The maximum distance allowed to be behind PIM (for entire transit).

F_i – The launch and recover speed per minute for cycle i.

V_s – The ordered sprint speed per minute during the dash between cycles.

V_{max} – The maximum permissible sprint speed per minute between flight operations periods.

V_{min} – The minimum speed per minute used in transit by the CVBG.

H_j – The minimum hours required flown by aircraft type .

EXPDIST $_i$ – The expected speed used in transit by the CVBG.

EPSON – The allowed deviation from EXPDIST_i allowed for each cycle.

$D_{1,i}$ – The distance traveled during cycle i relative to the CV's course from the CV position at the start of cycle i during the turn into the wind.

$D_{2,i}$ – The distance traveled during the launch and recovery phase of cycle i relative to the CV's course at the start of cycle i. $D_{2,i}$ is dependent upon the number of aircraft launched and recovered during cycle i.

$$D_{2,i} = D'_{2,i}(\text{launch/recovery time}_i)$$

$D'_{2,i}$ – A known multiple of the time to launch and recover aircraft during cycle i. $D'_{2,i}$ is used with the decision variables to obtain the distance traveled in phase 2 of cycle i.

$D_{3,i}$ – The distance traveled during cycle i relative to the CV's course from the CV position at the start of cycle i during the CV's turn to intercept a position on the PIM.

$D_{4,i}$ – The distance traveled during the intercycle sprint phase of cycle i relative to the CV's course at the start of cycle i. $D_{4,i}$ is dependent upon the number of aircraft launched and recovered during cycle i.

$$\begin{array}{c}\text{Intercycle}\\\text{sprint}\\\text{time}\end{array} = \begin{array}{c}\text{length}\\\text{of cycle}\end{array} - \begin{array}{c}\text{time to}\\\text{turn into}\\\text{the wind}\end{array} - \begin{array}{c}\text{time to}\\\text{turn back}\end{array} - \begin{array}{c}\text{time to}\\\text{launch}\\\text{and recover}\end{array}$$

$$D_{4,i} = D'_{4,i}\ (\text{intercycle sprint time})$$

$D'_{4,c}$ – A known multiple of the intercycle sprint time during cycle i. $D_{4,i}$ is used with the decision variables to obtain the distance traveled in phase 4 of cycle i.

P_i – The correction applied to the sum $D_{1,i},+D_{2,i}+D_{3,i}+D_{4,i}$ (which is the distance the CV has traveled relative to the CV's course at the start of cycle i) to determine the actual distance traveled relative to the PIM course.

$$P_i = \cos(\text{PIM course} - \text{CV course})$$

P_{C+1} – The correction applied to the sum $D_{1,C+1}+D_{2,C+1}+D_{3,C+1}$ to determine the distance traveled relative to PIM during the last recovery.

P_{sprint} – The correction applied to the sprint distance after flight operations to determine the distance traveled relative to PIM.

PIM – The actual distance to travel between flight operations periods or between the start of flight operations and a position which must be met.

$\$_{\#}$ – The penalty coefficient associated with an elastic variable $S\#$ not equal to 0. The larger the penalty, the less likely the constraint will be softened.

$MS_{i,j}$ – The maximum number of sorties allowed to launch of aircraft type j during cycle i.

TS_i – The maximum number of all aircraft sorties allowed to be launched during cycle c.

C – The number of cycles schedule for the flight operations period.

3. Constraint Coefficient Computations

One of the more critical constraints in the formulation of the LP model is the distance of travel along PIM that the CV needs to accomplish during the flight operations period. This constraint is extremely dependent upon initial estimates of the decision variables. Since the objective of the model is to determine the value of the decision variables which will maximize the sorties launched, the values of the coefficients used in the constraint to determine the distance traveled relative to PIM must first be approximated prior to execution of the model.

The coefficients themselves are functions of the turns the carrier conducts when turning into the wind and back to intercept PIM and are approximated by initially executing the flight analyzer option of COLP with estimates of the number of aircraft to be launched and recovered during each of the flight operations cycles. The estimates are found by determining the number of aircraft sorties which will be needed to meet the minimum tactical requirements. The analyzer will then track the CV through the flight operations period using these inputs to establish approximations for the turning angles required to turn into the wind for a favorable launch and recovery and to turn back to the PIM track after the launch and recovery cycle has been completed.

The turning angle will be applied to a transformation algorithm to provide relative PIM movement coefficients for a linear approximation to the distance traveled constraint. This transformation algorithm determines the distance relative to PIM the CV will travel during the four phases of each launch and recovery cycle. The phases are: (1) turn into the wind; (2) launch and recovery; (3) turn back to a course to intercept PIM; and (4) travel along intercept course. The distances traveled during each of these four phases are relative to the direction of travel at the beginning of the cycle prior to the CV's turn into the wind. This direction is the intercept course of phase 4 of the previous cycle. The distance traveled relative to PIM is obtained by multiplying the distance traveled during each cycle parallel to the last intercept course by the cosine of the angular offset between the actual PIM course and the intercept course. Figure 2 shows the four phases of the launch/recovery cycle and the distances of relative travel with respect to cycle course and PIM course.

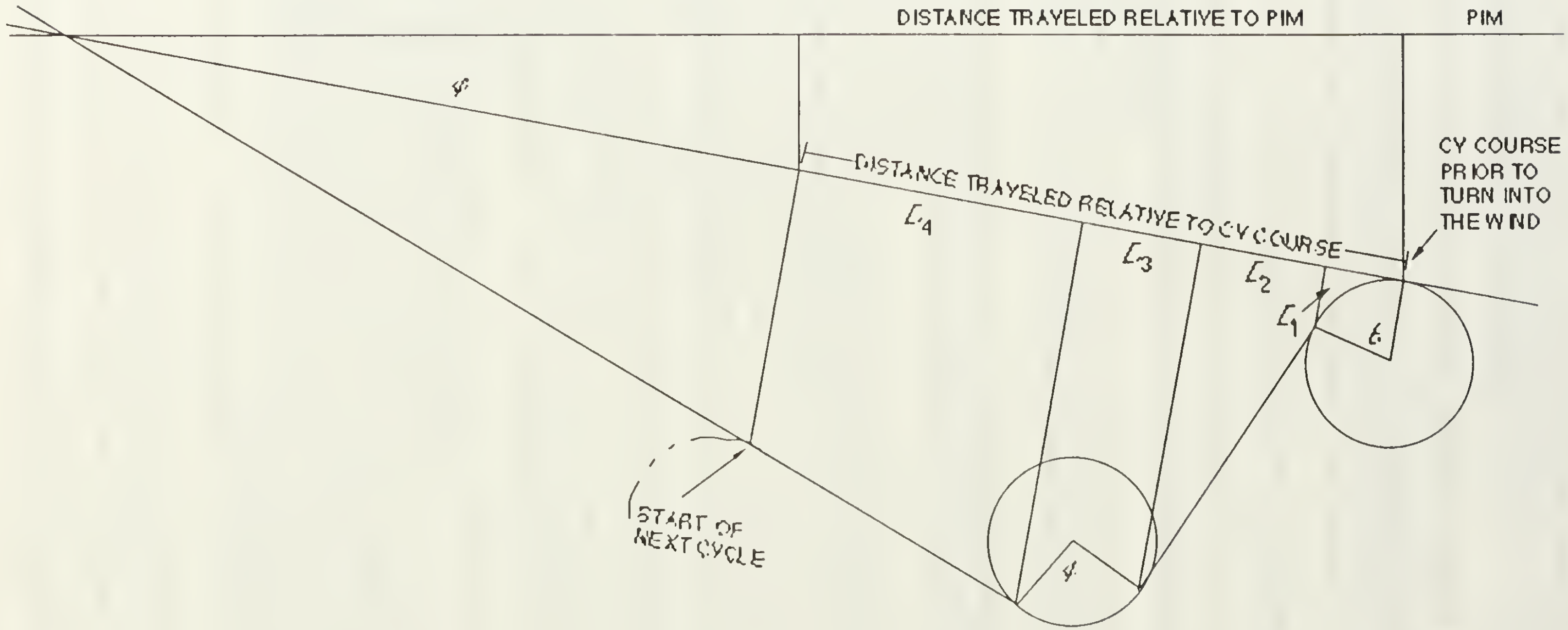

DISTANCE TRAVELED RELATIVE TO PIM
PIM
CY COURSE PRIOR TO TURN INTO THE WIND
DISTANCE TRAVELED RELATIVE TO CY COURSE
L_1
L_2
L_3
L_4
START OF NEXT CYCLE

Figure 2

To find the distance traveled relative to the CV course at the start of the flight cycle, the relative distance for each phase needs to be determined. If $D_{i,p}$ is the relative distance traveled during phase p of cycle i, then the total distance relative to initial course for cycle i is:

$$\sum_{p=1}^{4} D_{i,p}\text{, for each of the } C \text{ cycles.}$$

Figure 3 shows the travel of multiple launch/recovery cycles relative to the CV course at the start of the cycle. Let θ_i be the turning angle required to establish a course into the wind and let ϕ_i be the turning angle necessary after launch and recovery to turn to a course which will intercept PIM. The equations for

$D_{i,1}$ = (turning radius)sin θ_i

$D_{i,2}$ = (launch speed per minute)(time to launch and recover + lag time)cos θ_i

There are two evaluations possible for both $D_{i,3}$ and $D_{i,4}$ depending upon the direction of turn into the wind and the direction of turn back from the wind.

If both turns are made in the same direction:

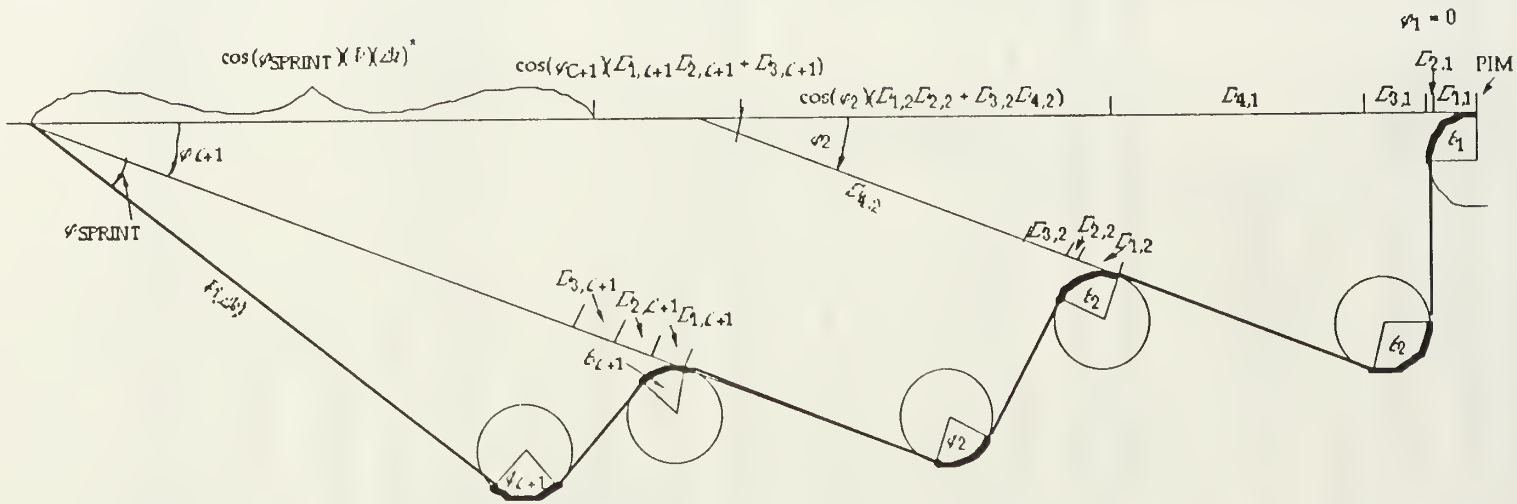
$D_{i,3}$ = (turn radius)[$\sin(\theta_i + \phi_i) - \sin(\theta_i)$]

$D_{i,4}$ = (intercycle sprint velocity per minute)[cycle i length–time to turn into the wind for cycle i – time to turn back for cycle i – (time to launch and recover in cycle i + lag time for cycle i)]$\cos(\phi_i + \theta_i)$

If both turns are made in opposite directions:

$D_{i,3}$ = (turning radius)[$\sin(\phi_i - \theta_i) + \sin\theta_i$]

$D_{i,4}$ = (intercycle sprint velocity per minute)[cycle i length–time to turn into the wind for cycle i – time to turn back for cycle i – (time to launch and recover in cycle i + lag time for cycle i)]$\cos(\phi_i - \theta_i)$



for the model to have a solution, there must be a V such that

(1) $V_m \le V \le V_s$

(2) $V(\Delta t)\cos(\varphi SPRINT)$ will cover the remaining distance

t, ψ, φ are only approximates based upon a trial execution of COLP t_1 is the only angle known for sure.

Figure 3

Once the distance traveled relative to CV course is computed, it is then adjusted to the distance traveled relative to PIM by a factor equal to the cosine of the difference between CV course to intercept and the PIM course (φ_i).

$$\text{Relative PIM Distance} = \cos(\varphi_i) \sum_{p=1}^{4} D_{i,p}$$

Total relative distance for the flight operation period is then found by summing the relative PIM distances over all cycles plus the relative PIM distance traveled during the last recovery plus the relative distance the CV could sprint during the time remaining until the next scheduled evolution (Figure 3).

To apply this linear approximation for total distance traveled to the LP model, modifications have been made to the values for $D_{i,2}$ and $D_{i,4}$. Both of these are dependent upon the number of aircraft launched and recovered during cycle i, and as previously mentioned, the number of aircraft launched and recovered is based upon the decision variables for the model. The coefficients used are then modified not to include the decision variables.

$D_{i,2}$ = $D'_{i,2}$ (Time to launch and recover + lag time)

where

$D'_{i,2}$ = (launch speed per minute)$\cos\theta_i$

and

$D_{i,4}$ = $D'_{i,4}$ [cycle i length – time to turn into the wind for cycle i – time to turn back for cycle i – (time to launch and recover in cycle i + lag time for cycle i)]

where $D'_{i,4}$

= (intercycle sprint velocity per minute) $\cos(\phi_i + \theta_i)$

= $V_s\cos(\phi_i + \theta_i)$,

if the turns are in the same direction or,

= (intercycle sprint velocity per minute)$\cos(\phi_i - \theta_i)$

= $V_s\cos(\phi_i - \theta_i)$,

if the turns are in the opposite direction. $D_{i,1}$, $D'_{i,2}$, $D_{i,3}$, and $D'_{i,4}$ are then used as coefficients in the distance constraint equation.

This linear approximation to distance traveled is fairly accurate if the turning angle is small or extremely large (the rate of change in the value of the cosine of an angle increases as the angle increases from 0 to 90 degrees and vice-versa as the angle increases from 90 to 180 degrees) or if the initial estimates for the turning angles are accurate. However, as the turn increases in size or the accuracy of the initial turning angles estimate becomes less accurate, the approximation to distance traveled incurs greater error. Since this model is used as a decision aid and there are a number of unforeseen circumstances which could occur that would change conditions under which flight operations are conducted, the error encountered in the linear approximation is acceptable.

4. Model Development

Maximize the number of tactical aircraft sorties launched and recovered at the carrier.

$$\max \sum_{i=1}^{C} \sum_{j=1}^{7} \sum_{k=1}^{3} x_{i,j,k}$$

Subject to

a. Do not exceed the maximum allowable number of sorties per cycle by aircraft type.

   The sum of the number of sorties of aircraft type j launched during cycle i for cycle lengths of 1, 2, or 3 cycles must not exceed the number of sorties authorized for launch of that aircraft type during cycle i.

$$\sum_{k=1}^{3} x_{i,j,k} \leq \mathrm{MS}_{i,j}, \qquad \begin{matrix} i = 1, \ldots, C \\ j = 1, \ldots, 7. \end{matrix}$$

Example: An F-14 squadron commanding officer desired that no more than six F-14 sorties be launched for each cycle.

$$\sum_{k=1}^{3} x_{i,j,k} \leq 6, \quad i = 1, \ldots, C.$$

b. Do not exceed an allowable maximum number of sorties per cycle.

The number of tactical sorties of all aircraft launched during i for cycle lengths 1, 2, or 3 must not exceed the number of sorties authorized for that cycle.

$$\sum_{j=1}^{7} \sum_{k=1}^{3} x_{i,j,k} \leq \mathrm{TS}_i, \quad i = 1, \ldots, C.$$

Example: An air group commander desires no more than 25 tactical launches per cycle.

$$\sum_{j=1}^{7} \sum_{k=1}^{3} x_{i,j,k} < 25, \qquad i = 1, \ldots, C.$$

c. Aircraft launched and recovered at the carrier must meet the airborne cycle requirement in addition to any specific mission tasking.

The combination of different length sorties of aircraft type j from the previous two cycles plus the sorties launched for the present cycle must meet the number of aircraft required to be airborne for tasking other than that assigned to meet specific mission requirements. Specific mission tasking which is input as constants to the model must be incorporated into this constraint to ensure a specifically tasked aircraft does not get counted as an additional airborne requirement. Thus when determining the number of aircraft to be airborne, the specific sorties must also be counted as aircraft required to be in the air during the cycles they are conducting their missions.

Specific tasked mission aircraft launched 2 cycles earlier for a length of three cycles which are airborne during the present cycle are:

$$\mathrm{FR}_{i-2,j,3}$$

Specific tasked mission aircraft launched one cycle earlier for a length of both 2 and 3 cycles which are airborne during the present cycle are:

$$\sum_{k=2}^{3} \mathrm{FR}_{i-1,j,k}.$$

Specific tasked mission aircraft to be launched in the present cycle are:

$$\sum_{k=1}^{3} \mathrm{FR}_{i,j,k}.$$

Miscellaneous or other mission requirements for aircraft of type j needed airborne during cycle i are:

$$OR_{i,j}.$$

Tactical sorties launched in cycle i to be airborne are of cycle lengths 1, 2, and 3 and must be sufficient to meet the additional aircraft needed airborne in cycle i, $i+1$, $i+2$, plus the specific mission sorties scheduled for launch in cycle i.

$$\sum_{k=1}^{3} x_{i,j,k}.$$

Tactical sorties launched in cycle $i-1$ remaining airborne are those of cycle lengths 2 and 3 which can be used to meet the airborne requirements for cycles i and $i+1$, and the specific mission requirements for two or three cycle length missions launched in cycle $i-1$.

$$\sum_{k=2}^{3} x_{i-1,j,k}.$$

Tactical sorties launched in cycle $i-2$ remaining airborne are those of three cycle length which can be used to meet the airborne requirement for cycles $i-2$, $i-1$ and i, and the specific mission sorties launched in cycle $i-2$ on a three cycle mission.

$$x_{i-2,j,3}.$$

The addition airborne mission requirement can be met by any combination of single, double or triple cycle launches provided that the aircraft needed for the specific mission tasking are available.

$$\sum_{k=1}^{3} x_{i,j,k} + \sum_{k=2}^{3} x_{i-1,j,k} + x_{i-2,j,3} \geq AR_{i,j}, \qquad \begin{array}{l} i = 1,\ldots,C \\ j = 1,\ldots,7 \end{array}$$

where

$$AR_{i,j} = \sum_{k=1}^{3} FR_{i,j,k} + \sum_{k=2}^{3} FR_{i-1,j,k} + FR_{i-2,j,3} + OR_{ij}$$

Example: In addition to the fixed requirement to launch four A-6s on cycle 3 for a strike mission, there is a need to have two A-6s airborne during cycle 3 for surface surveillance. These two extra A-6s could be from a triple cycle launch in cycle 1, a double or triple launch in cycle 2, or a launch of cycle lengths 1, 2, or 3 in cycle 3.

$$\begin{aligned} AR_{3,3} &= FR_{3,3,2} + OR_{3,3} \\ &= 4 + 2 \\ &= 6 \end{aligned}$$

$$\sum_{k=1}^{3} x_{3,3,k} + \sum_{k=2}^{3} x_{2,3,k} + x_{1,3,3} \geq 6.$$

d. Cannot exceed the aircraft availability per cycle.

The number of aircraft type j to launch in cycle i cannot exceed the number of aircraft type j available in cycle i. The aircraft of type j available for cycle i are those aircraft which are not presently airborne or recovering during cycle i. Aircraft airborne or recovering in cycle i are the triple cycle aircraft launched in the previous three cycles, the double cycle aircraft launched in the previous two cycles, the single cycle aircraft launched in the previous cycle, the launches of all lengths for the present cycle, and the non-returning aircraft launched in all previous cycles.

$$x_{i,j,0} + \sum_{k=1}^{3} \left(x_{i,j,k} + x_{i-1,j,k}\right) + \sum_{k=2}^{3} x_{i-2,j,k} + x_{i-3,j,3} + \sum_{l=1}^{i} x_{l,j,4} \leq AC_{i,j}, \qquad \begin{array}{l} i = 1,\ldots,C \\ j = 1,\ldots,7. \end{array}$$

However, when the value for the length of a cycle is equal to −1, 0, or 4 we are working with fixed requirements and can thus display those sorties as a fixed variable instead of a decision variable:

$$x_{i,j,-1} = \mathrm{FR}_{i,j,-1}$$

$$x_{i,j,0} = \mathrm{FR}_{i,j,0}$$

$$x_{i,j,4} = \mathrm{FR}_{i,j,4}$$

$$FR_{i,j,0} + \sum_{k=1}^{3}\left(x_{i,j,k} + x_{i-1,j,k}\right) + \sum_{k=2}^{3} x_{i-2,j,k} + x_{i-3,j,3} + \sum_{l=1}^{i} \mathrm{FR}_{i,j,4} \leq \mathrm{AC}_{i,j}, \quad \begin{array}{l} i = 1,\dots,C \\ j = 1,\dots,7. \end{array}$$

Rewriting to keep all constants on the right hand side gives:

$$\sum_{k=1}^{3}\left(x_{i,j,k} + x_{i-1,j,k}\right) + \sum_{k=2}^{3} x_{i-2,j,k} + x_{i-3,j,3} \leq \mathrm{AC}_{i,j} - \mathrm{FR}_{i,j,0} - \sum_{l=1}^{i} \mathrm{FR}_{i,j,4}, \quad \begin{array}{l} i = 1,\dots,C \\ j = 1,\dots,7. \end{array}$$

e. Must meet minimum flight hour requirements for each type of aircraft.

Due to squadron hour and training requirements, each aircraft type will need to fly a minimum number of hours during each flight operations period. The sum of the time flown by each sortie of aircraft type j must meet or exceed the minimum number required.

$$\sum_{i=1}^{C}\left[(LC_i)x_{i,j,1} + (LC_i + LC_{i-1})x_{i-1,j,2} + (LC_i + LC_{i-1} + LC_{i-2})x_{i-2,j,2}\right]$$

$$\geq H_j,\ j = 1,\dots,7.$$

f. Cannot exceed cycle time.

One of the more important constraints in the model is the time allotted each launch and recovery cycle. In this time, the aircraft carrier must be able to turn into the wind, launch and recover aircraft, respot/rearm aircraft for the next cycle, and then turn to a course to intercept PIM track. If any time remains in the cycle, after the above has been accomplished, the carrier can sprint in an attempt to recover if behind PIM.

$$\begin{array}{c}\text{time to}\\ \text{turn into}\\ \text{the wind}\end{array} + \begin{array}{c}\text{launch}\\ \text{time}\end{array} + \begin{array}{c}\text{recovery}\\ \text{time}\end{array} + \begin{array}{c}\text{lag}\\ \text{time}\end{array} + \begin{array}{c}\text{time to turn}\\ \text{to intercept}\\ \text{PIM}\end{array} + \begin{array}{c}\text{respot/}\\ \text{rearm}\\ \text{time}\end{array} + \begin{array}{c}\text{sprint}\\ \text{time}\end{array} = \begin{array}{c}\text{length}\\ \text{of the}\\ \text{cycle}\end{array}$$

Since sprint time is actually part of the solution (the more time in the launch and recovery phase, the less time to make up for lost distance) and only exists if there is time after all other evolutions in the cycle have been completed, the constraint can be rewritten as:

$$\begin{matrix}\text{time to} & & & & & & & & \text{time to turn} & & \text{respot/} & & \text{length}\\ \text{turn into} & + & \text{launch time} & + & \text{recovery time} & + & \text{lag time} & + & \text{to intercept} & + & \text{rearm} & \leq & \text{of the}\\ \text{the wind} & & & & & & & & \text{PIM} & & \text{time} & & \text{cycle}\end{matrix}$$

Defining:

$$\text{launch time}_i = LT_i(\#\ \text{of launches}_i)$$

$$= LT_i \sum_{j=1}^{9} \sum_{k=0}^{4} x_{i,j,k}$$

$$= LT_i \left[\sum_{j=1}^{9} \left(FR_{i,j,0} + FR_{i,j,4} \right) + \sum_{j=8}^{9} \sum_{k=1}^{3} FR_{i,j,k} + \sum_{j=1}^{7} \sum_{k=1}^{3} x_{i,j,k} \right]$$

$$\text{recovery time}_i = RT_i(\#\ \text{of recoveries}_i)$$

$$= RT_i \left[\sum_{j=1}^{9} \left(x_{i,j,0} + x_{i,j,-1} + x_{i-1,j,1} + x_{i-2,j,2} + x_{i-3,j,3} \right) \right]$$

$$= RT_i \left[\sum_{j=1}^{9} \left(FR_{i,j,0} + FR_{i,j,-1} \right) + \sum_{j=8}^{9} \left(FR_{i-1,j,1} + FR_{i-2,j,2} + FR_{i-3,j,3} \right) \right.$$

$$\left. + \sum_{j=1}^{7} \left(x_{i-1,j,1} + x_{i-2,j,2} + x_{i-3,j,3} \right) \right].$$

$$\text{lag time}_i = LG_i = K_0 + K_1(\#\ \text{of launches}_i).$$

$$\text{respot/rearm time}_i = RS_i(\#\text{ of recoveries}_i)$$

$$= RS_i\left[\sum_{j=1}^{9}\left(x_{i,j,0} + x_{i,j,-1} + x_{i-1,j,1} + x_{i-2,j,2} + x_{i-3,j,3}\right)\right]$$

$$= RS_i\left[\sum_{j=1}^{9}\left(FR_{i,j,0} + FR_{i,j,-1}\right) + \sum_{j=8}^{9}\left(FR_{i-1,j,1} + FR_{i-2,j,2} + FR_{i-3,j,3}\right)\right.$$

$$\left. + \sum_{j=1}^{7}\left(x_{i-1,j,1} + x_{i-2,j,2} + x_{i-3,j,3}\right)\right]$$

Fixed requirements are substituted for sorties representing a nonoriginating recovery, yo-yo launch/recovery, and nonreturning launches (cycle lengths of –1, 0, or 4).

Using the above equations for launch, recovery and lag times, and time to turn into the wind and to regain PIM, the constraint equation for cycle time is:

$$LT_i\left[\sum_{j=1}^{9}\left(FR_{i,j,0} + FR_{i,j,4}\right) + \sum_{j=8}^{9}\sum_{k=1}^{3} FR_{i,j,k} + \sum_{j=1}^{7}\sum_{k=1}^{3} x_{i,j,k}\right]$$

$$+(RT_i + RS_i)\left[\sum_{j=1}^{9}\left(FR_{i,j,0} + FR_{i,j,-1}\right) + \sum_{j=8}^{9}\left(FR_{i-1,j,1} + FR_{i-2,j,2} + FR_{i-3,j,3}\right)\right.$$

$$\left. + \sum_{j=1}^{7}\left(x_{i-1,j,1} + x_{i-2,j,2} + x_{i-3,j,3}\right)\right]$$

$$+ K_0 + K_1\left[\sum_{j=1}^{9}\left(FR_{i,j,0} + FR_{i,j,4}\right) + \sum_{j=8}^{9}\sum_{k=1}^{3} FR_{i,j,k} + \sum_{j=1}^{7}\sum_{k=1}^{3} x_{i,j,k}\right]$$

$$+ TI_i + TO_i \le LC_i, i = 1, \ldots, C.$$

Finally, rearranging and moving all constants and fixed values to the right hand side yields:

$$(LT_i + K_1)\left[\sum_{j=1}^{7}\sum_{k=1}^{3} x_{i,j,k}\right] + (RT_i + RS_i)\sum_{j=1}^{7}\left(x_{i-1,j,1} + x_{i-2,j,2} + x_{i-3,j,3}\right)$$

$$\leq LC_i - TI_i - TO_i - K_0$$

$$-(LT_i + K_1)\left[\sum_{j=1}^{9}\left(FR_{i,j,0} + FR_{i,j,4}\right) + \sum_{j=8}^{9}\sum_{k=1}^{3} FR_{i,j,k}\right]$$

$$-(RT_i + RS_i)\left[\sum_{j=1}^{9}\left(FR_{i,j,0} + FR_{i,j,-1}\right) + \sum_{j=8}^{9}\left(FR_{i-1,j,1} + FR_{i-2,j,2} + FR_{i-3,j,3}\right)\right],$$

$$i = 1, \ldots, C.$$

g. Must meet minimum and maximum distance to travel along PIM.

Probably the most important restriction placed upon the optimization model is the requirement to be at or within some distance of a point on the PIM at a specific time. Reasons for being at this point may be an underway replenishment rendezvous, sanitation area rendezvous, or rendezvous to be on PIM for the start of the next scheduled battle group evolution.

Since the CV must deviate from PIM track in order to conduct flight operations, its projected progress along the original PIM track must permit it to arrive at a specific PIM point for the start of the next battle group evolution. The projected travel along the PIM track would normally require the use of nonlinear equations as a result of calculating distance in a Cartesian coordinate system, and a decision process to determine which turning directions are needed to turn into the wind and back to intercept PIM. However, through some modification, linear equations can be used to approximate the projected travel along the PIM. This distance will correspond exactly with the true distance if there is no requirement to turn into the wind or the estimated turn angles are correct. Otherwise, as the number of turns or the size of the turns increases, the distance is approximate. Since this model is designed to be a tactical decision aid, the accuracy lost as a result of the approximation does not significantly degrade the solution sufficiently to justify a more complex or time costly model.

	−	≤	+	+	≤	−
PIM distance to travel	distance permitted to be short of PIM point of next evolution	sum of the distances traveled relative to PIM for each cycle	the distance traveled during the final recovery	the distance traveled from the end of the flight operations period until the next scheduled evolution	PIM distance to travel	distance permitted to be ahead of PIM point of next evolution

Minimum distance to travel is (PIM distance–distance behind allowed) using the maximum sprint velocity.

Maximum distance to travel is (PIM distance+distance ahead allowed) using the minimum sprint velocity.

Distance relative to PIM traveled during each cycle i

$$= P_i\left(D_{1,i} + D_{2,i} + D_{3,i} + D_{4,i}\right), i = 1, \ldots, C$$

$$= P_i\left\{D_{1,i} + D'_{2,i}\left[LT_i(\text{\# of launches}_i) + RT_i(\text{\# of recoveries}_i) + LG_i\right] + D_{3,i}\right.$$
$$\left. + D'_{4,i}\left(LC_i - TI_i - TO_i - \left[LT_i(\text{\# of launches}_i) + RT_i(\text{\# of recoveries}_i) + LG_i\right]\right)\right\}$$

$$= P_i\left\{D_{1,i} + D_{3,i} + \left(D'_{2,i} - D'_{4,i}\right)\left[LT_i(\text{\# of launches}_i) + RT_i(\text{\# of recoveries}_i) + LG_i\right]\right.$$
$$\left. + D'_{4,i}(LC_i - TI_i - TO_i)\right\}, i = 1, \ldots, C.$$

Distance relative to PIM traveled over all cycles

$$= \sum_{i=1}^{C}\left[P_i \left\{D_{1,i} + D_{3,i} + \left(D'_{2,i} - D'_{4,i}\left[LT_i(\text{\# of launches}_i) + RT_i(\text{\# of recoveries}_i) + LG_i\right]\right.\right.\right.$$
$$\left.\left. + D'_{4,i}(LC_i - TI_i - TO_i)\right\}\right]$$

Distance traveled relative to PIM on final recovery

$$= P_{C+1}\left\{D_{1,C+1} + D_{3,C+1} + D'_{2,C+1}\left[LT_{C+1}(\text{\# of launches}_{C+1}) + RT_{C+1}\right]\right.$$
$$\left.(\text{\# of recoveries}_{C+1}) + LG_{C+1}\right\}$$

Distance traveled relative to PIM after returning to a course to intercept PIM following last recovery until the start of the next evolution

$$= \left(P_{\text{sprint}}\right)(\text{Sprint velocity per minute})(\text{Time sprinting})$$

$$= \left(P_{\text{sprint}}\right)(V)\left[\begin{pmatrix}\text{time from end of} \\ \text{last full cycle until} \\ \text{start of next evolution}\end{pmatrix} - \begin{pmatrix}\text{time required to conduct} \\ \text{final recovery and return} \\ \text{to intercept course}\end{pmatrix}\right]$$

$$= \left(P_{\text{sprint}}\right)(V)\left(DELTA - TI_{C+1} - TO_{C+1}\right.$$
$$\left. - \left[LT_{C+1}(\text{\# of launches}_{C+1}) + RT_{C+1}(\text{\# of recoveries}_{C+1}) + LG_{C+1}\right]\right)$$

$$** \; V = V_{max} = \text{Maximum sprint velocity}\begin{pmatrix}\text{Used by CV to meet minimum}\\ \text{distance to travel during the sprint}\end{pmatrix}$$

$$= V_{min} = \text{Minimum sprint velocity}\begin{pmatrix}\text{Used by CV to remain within maximum}\\ \text{distance allowed to travel during the sprint}\end{pmatrix}$$

The range (V_{min},V_{max}) implies that there exists a velocity at which the CV can sprint to meet the requirement to be within a given distance of the PIM rendezvous point.

Combining the different parts of travel relative to PIM and using the equation definitions for number of launches and recoveries, and lag time utilized in the cycle length constraints gives the total distance traveled relative to PIM from the start of flight operations until the start of the next planned evolution.

$$\sum_{i=1}^{C} [P_i \{D_{1,i} + D_{3,i} + (D'_{2,i} - D'_{4,i})[LT_i(\#\text{ of launches}_i) + RT_i(\#\text{ of recoveries}_i) + LG_i]$$

$$+ D'_{4,i}(LG_i - TI_i - TO_i)\}]$$

$$+ P_{C+1}\{D_{1,C+1} + D_{3,C+1} + D'_{2,C+1}[LT_{C+1}(\#\text{ of launches}_{C+1})$$

$$+ RT_{C+1}(\#\text{ of recoveries}_{C+1}) + LG_{C+1}]\}$$

$$+ (P_{\text{sprint}})(V)\{DELTA - TI_{C+1} - TO_{C+1}$$

$$- [LT_{C+1}(\#\text{ of launches}_{C+1}) + RT_{C+1}(\#\text{ of recoveries}_{C+1}) + LG_{C+1}]\}$$

Converting to an equation which uses the pre-defined variables gives:

$$= \sum_{i=1}^{C}\left[P_i \left\{ D_{1,i} + D_{3,i} \right.\right.$$

$$+ (D'_{2,i} - D'_{4,i})\left[LT_i \left[\sum_{j=1}^{9}(FR_{i,j,0} + FR_{i,j,4}) + \sum_{j=8}^{9}\sum_{k=1}^{3} FR_{i,j,k} + \sum_{j=1}^{7}\sum_{k=1}^{3} x_{i,j,k} \right]\right.$$

$$+RT_i\left[\sum_{j=1}^{9}\left(FR_{i,j,0}+FR_{i,j,-1}\right)+\sum_{j=8}^{9}\left(FR_{i-1,j,1}+FR_{i-2,j,2}+FR_{i-3,j,3}\right)\right.$$

$$\left.+\sum_{j=1}^{7}\left(x_{i-1,j,1}+x_{i-2,j,2}+x_{i-3,j,3}\right)\right]+K_0$$

$$+K_1\left[\sum_{j=1}^{9}\left(FR_{i,j,0}+FR_{i,j,4}\right)+\sum_{j=8}^{9}\sum_{k=1}^{3}FR_{i,j,k}+\sum_{j=1}^{7}\sum_{k=1}^{3}x_{i,j,k}\right]\Bigg]$$

$$\left.\left.+D'_{4,i}\ \left(LC_i-TI_i-TO_i\right)\right\}\right]$$

$$+P_{C+1}\left\{D_{1,C+1}+D_{3,C+1}+D'_{2,C+1}\left[LT_{C+1}\sum_{j=1}^{9}\left(FR_{C+1,j,0}+FR_{C+1,j,4}\right)\right.\right.$$

$$+RT_{C+1}\left[\sum_{j=1}^{9}\left(FR_{C+1,j,0}+FR_{C+1,j,-1}\right)\right.$$

$$\left.+\sum_{j=8}^{9}\left(FR_{C,j,1}+FR_{C-1,j,2}+FR_{C-2,j,3}\right)+\sum_{j=1}^{7}\left(x_{C,j,1}+x_{C-1,j,2}+x_{C-2,j,3}\right)\right]$$

$$\left.\left.+K_0+K_1\left[\sum_{j=1}^{9}\left(FR_{C+1,j,0}+FR_{C+1,j,4}\right)\right]\right]\right\}$$

$$+\left(P_{\text{sprint}}\right)(V)\left\{DELTA-TI_{C+1}-TO_{C+1}-\left[LT_{C+1}\sum_{j=1}^{9}\left(FR_{C+1,j,0}+FR_{C+1,j,4}\right)\right.\right.$$

$$+RT_{C+1}\left[\sum_{j=1}^{9}\left(FR_{C+1,j,0}+FR_{C+1,j,-1}\right)+\sum_{j=8}^{9}\left(FR_{C,j,1}+FR_{C-1,j,2}+FR_{C-2,j,3}\right)\right.$$

$$\left.\left.+\sum_{j=1}^{7}\left(x_{C,j,1}+x_{C-1,j,2}+x_{C-2,j,3}\right)\right]+K_0+K_1\sum_{j=1}^{9}\left(FR_{C+1,j,0}+FR_{C+1,j,4}\right)\right]\right\}$$

which must be bounded by (PIM – B) on the lower end and (PIM + A) on the upper end.

Placing all constants and user entered variables on the right hand side and splitting into two constraints; one for a minimum distance to travel and one for the maximum distance yields:

$$\sum_{i=1}^{C}\left\{P_i\left(D'_{2,i}-D'_{4,i}\right)\left[\left(LT_i+K_1\right)\sum_{j=1}^{7}\sum_{k=1}^{3}x_{i,j,k}+RT_i\sum_{j=1}^{7}\left(x_{i-1,j,1}+x_{i-2,j,2}+x_{i-3,j,3}\right)\right]\right\}$$

$$+\left(P_{C+1}D'_{2,C+1}-P_{sprint}V_{max}\right)RT_{C+1}\sum_{j=1}^{7}\left(x_{C,j,1}+x_{C-1,j,2}+x_{C-2,j,3}\right)$$

$$\geq PIM-B-\sum_{i=1}^{C}\left\{P_i\left[D_{1,i}+D_{3,i}+D'_{4,i}\left(LC_i-TI_i-TO_i\right)+\left(D'_{2,i}-D'_{4,i}\right)\left\{K_0+\left(LT_i+K_1\right)\right.\right.\right.$$

$$\left[\sum_{j=1}^{9}\left(FR_{i,j,0}+FR_{i,j,4}\right)+\sum_{j=8}^{9}\sum_{k=1}^{3}FR_{i,j,k}\right]$$

$$\left.\left.\left.+RT_i\left[\sum_{j=1}^{9}\left(FR_{i,j,0}+FR_{i,j,-1}\right)+\sum_{j=8}^{9}\left(FR_{i-1,j,1}+FR_{i-2,j,2}+FR_{i-3,j,3}\right)\right]\right\}\right]\right\}$$

$$-P_{C+1}\left(D_{1,C+1}+D_{3,C+1}+K_0D'_{2,C+1}\right)-P_{sprint}V_{max}\left(DELTA-TI_{C+1}-TO_{C+1}-K_0\right)$$

$$-\left(P_{C+1}D'_{2,C+1} - P_{\text{sprint}}V_{\max}\right)\left\{(LT_{C+1}+K_1)\sum_{j=1}^{9}\left(FR_{C+1,j,0}+FR_{C+1,j,4}\right)\right.$$

$$\left.+RT_{C+1}\left[\sum_{j=1}^{9}\left(FR_{C+1,j,0}+FR_{C+1,j,-1}\right)+\sum_{j=8}^{9}\left(FR_{C,j,1}+FR_{C-1,j,2}+FR_{C-2,j,3}\right)\right]\right]\Bigg\}$$

and

$$\sum_{i=1}^{C}\left\{P_i\left(D'_{2,i}-D'_{4,i}\right)\left[(LT_i+K_1)\sum_{j=1}^{7}\sum_{k=1}^{3}x_{i,j,k}+RT_i\sum_{j=1}^{7}\left(x_{i-1,j,1}+x_{i-2,j,2}+x_{i-3,j,3}\right)\right]\right]\Bigg\}$$

$$+\left(P_{C+1}D'_{2,C+1}-P_{\text{sprint}}V_{\min}\right)RT_{C+1}\sum_{j=1}^{7}\left(x_{C,j,1}+x_{C-1,j,2}+x_{C-2,j,3}\right)$$

$$\leq PIM + A - \sum_{i=1}^{C}\left\{P_i\left[D_{1,i}+D_{3,i}+D'_{4,i}(LC_i - TI_i - TO_i)\right.\right.$$

$$+\left(D'_{2,i}-D'_{4,i}\right)\left\{K_0+(LT_i+K_1)\left[\sum_{j=1}^{9}\left(FR_{i,j,0}+FR_{i,j,4}\right)+\sum_{j=8}^{9}\sum_{k=1}^{3}FR_{i,j,k}\right]\right.$$

$$\left.\left.\left.+RT_i\left[\sum_{j=1}^{9}\left(FR_{i,j,0}+FR_{i,j,-1}\right)+\sum_{j=8}^{9}\left(FR_{i-1,j,1}+FR_{i-2,j,2}+FR_{i-3,j,3}\right)\right]\right]\right]\right\}$$

$$-P_{C+1}\left(D_{1,C+1}+D_{3,C+1}+K_0D'_{2,C+1}\right)-P_{\text{sprint}}V_{\max}(DELTA - TI_{C+1} - TO_{C+1} - K_0)$$

$$-\left(P_{C+1}D'_{2,C+1}-P_{\text{sprint}}V_{\max}\right)\left\{(LT_{C+1}+K_1)\sum_{j=1}^{9}\left(FR_{C+1,j,0}+FR_{C+1,j,4}\right)+RT_{C+1}\right.$$

$$\left[\sum_{j=1}^{9}\left(FR_{C+1,j,0}+FR_{C+1,j,-1}\right)+\sum_{j=8}^{9}\left(FR_{C,j,1}+FR_{C-1,j,2}+FR_{C-2,j,3}\right)\right]\Bigg]\Bigg\}$$

h. Restrict the distance behind PIM at the end of each cycle.

The BG commander, as an option, may desire to remain within a specific distance of PIM at the endof each cycle. This constraint can be easily incorporated into the model by requiring the distance traveled relative to PIM meet or exceed a specific minimum determined by the scheduled PIM speed and allowable deviation for all cycles up to the present cycle.

Applying the equation for distance traveled relative to PIM during each cycle which was developed in constraint g, the following can be developed.

Distance relative to PIM traveled up to cycle i =

$$\sum_{l=1}^{i}\Big[P_l\Big\{D_{1,l}+D_{3,l}+\left(D'_{2,l}-D'_{4,l}\right)\left[LT_l(\text{\# of launches}_l)+RT_l(\text{\# of recoveries}_l)+LG_l\right]$$

$$+D'_{4,l}\left(LC_l-TI_l-TO_l\right)\Big\}\Big]$$

Now constraining the distance:

$$\sum_{l=1}^{i}\Bigg[P_l\Bigg\{D_{1,l}+D_{3,l}\left(D'_{2,l}-D'_{4,l}\right)\Bigg[LT_l\left[\sum_{j=1}^{9}\left(FR_{l,j,0}+FR_{l,j,4}\right)+\sum_{j=8}^{9}\sum_{k=1}^{3}FR_{l-1,j,1}+\sum_{j=1}^{7}\sum_{k=1}^{3}x_{l,j,k}\right]$$

$$+RT_l\left[\sum_{j=1}^{9}\left(FR_{l,j,0}+FR_{l,j,-1}\right)+\sum_{j=8}^{9}\left(FR_{l-1,j,1}+FR_{l-2,j,2}+FR_{l-3,j,3}\right)+\sum_{j=1}^{7}\left(x_{l-1,j,1}+x_{l-2,j,2}+x_{l-3,j,3}\right)\right]$$

$$+K_0+K_1\left[\sum_{j=1}^{9}\left(FR_{l,j,0}+FR_{l,j,4}\right)+\sum_{j=8}^{9}\sum_{l=1}^{3}FR_{l,j,k}+\sum_{j=1}^{7}\sum_{k=1}^{3}x_{l,j,k}\right]\Bigg]+D'_{4,l}\left(LC_l-TI_l-TO_l\right)\Bigg\}\Bigg]$$

$$<\sum_{l=1}^{i}EXPDIST_l-EPSON, \qquad \forall i$$

Finally,

$$\sum_{l=1}^{i}\left\{P_l\left(D'_{2,l}-D'_{4,l}\right)\left[\left(LT_l+K_1\right)\sum_{j=1}^{7}\sum_{k=1}^{3}x_{l,j,k}+RT_l\sum_{j=1}^{7}\left(x_{l-1,j,1}+x_{l-2,j,2}+x_{l-3,j,3}\right)\right]\right\}$$

$$<\sum_{l=1}^{i}EXPDIST_l-EPSON-\sum_{l=1}^{i}\left\{P_l\left[D_{1,l}+D_{3,l}+D'_{4,l}\left(LC_l-TI_l-TO_l\right)+\left(D'_{2,l}-D'_{4,l}\right)\right.\right.$$

$$\left\{K_0+\left(LT_l+K_1\right)\left[\sum_{j=1}^{9}\left(FR_{l,j,0}+FR_{l,j,4}\right)+\sum_{j=8}^{9}\sum_{k=1}^{3}FR_{l,j,k}\right]+RT_l\left[\sum_{j=1}^{9}\left(FR_{l,j,0}+FR_{l,j,-1}\right)\right.\right.$$

$$\left.\left.\left.\left.+\sum_{j=8}^{9}\left(FR_{l-1,j,1}+FR_{l-2,j,2}+FR_{l-3,j,3}\right)\right]\right\}\right]\right\},\qquad\forall i$$

In addition, by placing the following bounds on the decision variables:

1. Meet minimum fixed requirements for the sorties of concern.

   The number of sorties of length k scheduled for aircraft type j must meet the tasking requirements for each cycle.

   $$x_{i,j,k}\geq FR_{i,j,k},\forall_{i,j,k}$$

   Example: The tasking of a four-plane strike mission requires it to launch at cycle 3 and recover at cycle 5.

   $$x_{3,3,2}\geq 4$$

2. Number of sorties must be not-negative.

   $$x_{i,j,k}\geq 0,\forall_{i,j,k}$$

3. Additional Bounds
   Some aircraft are unable to fly for the full lengths possible. Additionally, it makes no sense to launch an aircraft for a flight duration longer than the number of remaining cycles, thus:

   $$\left.\begin{array}{l}x_{i,j,1}=0\\x_{i,j,2}=0\\x_{i,j,3}=0\end{array}\right\};\quad\text{for selected combinations of } j \text{ and } i.$$

and

$$x_{C,j,2} = 0, \qquad \forall j$$
$$x_{C,j,3} = 0, \qquad \forall j$$
$$x_{C-1,j,3} = 0, \qquad \forall j$$

Finally, after incorporating the elastic variables into the appropriate constraints, the linear programming model used to find the maximum number of sorties which can be launched while meeting PIM transit requirements is:

$$\max \sum_{i=1}^{C}\sum_{j=1}^{7}\sum_{k=1}^{3} x_{i,j,k} - \sum_{i=1}^{C}\sum_{j=1}^{7}\left(\$_1 S1_{i,j} + \$_2 S2_{i,j}\right) - \sum_{i=1}^{C} \$_3 S3_i - \sum_{j=1}^{7} \$_4 S4_j - \$_5 S5$$
$$-\$_6 S6 \left\{ -\sum_{i=1}^{C} \$_7\, S7_i \right\}$$

Subject to

- $$\sum_{k=1}^{3} x_{i,j,k} - S2_{i,j} \le MS_{i,j}, \qquad \forall ij$$
- $$\sum_{j=1}^{7}\sum_{k=1}^{3} x_{i,j,k} - S3_i \le TS_i, \qquad \forall i$$
- $$\sum_{k=1}^{3} x_{i,j,k} + \sum_{k=2}^{3} x_{i-1,j,k} + x_{i-2,j,3} \ge AR_{i,j}, \qquad \forall ij$$
- $$\sum_{k=1}^{3}\left(x_{i,j,k} + x_{i-1,j,k}\right) + \sum_{k=2}^{3} x_{i-2,j,k} + x_{i-3,j,3} \le AC_{i,j} - \sum_{l=1}^{i} FR_{l,j,0} - \sum_{l=1}^{i} FR_{l,j,4}, \quad \forall ij$$

$$\sum_{i=1}^{C}\left[(LC_i)(x_{i,j,1}) + (LC_i + LC_{i-1})(x_{i-1,j,2}) + (LC_i + LC_{i-1} + LC_{i-2})(x_{i-2,j,3})\right] + S4_j$$
$$\ge H_j, \qquad \forall j$$

- $$(LT_i + K_1)\left(\sum_{j=1}^{7}\sum_{k=1}^{3} x_{i,j,k}\right) + (RT_i + RS_i)\sum_{j=1}^{7}\left(x_{i-1,j,1} + x_{i-2,j,2} + x_{i-3,j,3}\right) +$$

$$\leq LC_i - TI_i - TO_i - K_0 - (LT_i + K_1)\left[\sum_{j=1}^{9}\left(FR_{i,j,0} + FR_{i,j,4}\right) + \sum_{j=8}^{9}\sum_{k=1}^{3} FR_{i,j,k}\right]$$

$$-(RT_i + RS_i)\left[\sum_{j=1}^{9}\left(FR_{i,j,0} + FR_{i,j,-1}\right) + \sum_{j=8}^{9}\left(FR_{i-1,j,1} + FR_{i-2,j,2} + FR_{i-3,j,3}\right)\right], \ \forall i$$

- $$\sum_{i=1}^{C}\left\{P_i\left(D'_{2,i} - D'_{4,i}\right)\left[(LT_i + K_1)\sum_{j=1}^{7}\sum_{k=1}^{3} x_{i,j,k} + RT_i\sum_{j=1}^{7}\left(x_{i-1,j,1} + x_{i-2,j,2} + x_{i-3,j,3}\right)\right]\right\}$$

$$+\left(P_{C+1}D'_{2,C+1} - P_{\text{sprint}}V_{\max}\right)RT_{C+1}\sum_{j=1}^{7}\left(x_{C,j,1} + x_{C-1,j,2} + x_{C-2,j,3}\right) + S5$$

$$\geq PIM - B - \sum_{i=1}^{7}\left\{P_i\left[D_{1,i} + D_{3,i} + D'_{4,i}(LC_i - TI_i - TO_i) + \left(D'_{2,i} - D'_{4,i}\right)\right.\right.$$

$$\left\{K_0 + (LT_i + K_i)\left[\sum_{j=8}^{9}\left(FR_{i,j,0} + FR_{i,j,4}\right) + \sum_{j=8}^{9}\sum_{k=1}^{3} FR_{i,j,k}\right]\right.$$

$$\left.\left.\left.+ RT_i\left[\sum_{j=1}^{9}\left(FR_{i,j,0} + FR_{i,j,-1}\right) + \sum_{j=8}^{9}\left(FR_{i-1,j,1} + FR_{i-2,j,2} + FR_{i-3,j,3}\right)\right]\right\}\right]\right\}$$

$$-P_{C+1}\left(D_{1,C+1} + D_{3,C+1} + K_0 D'_{2,C+1}\right) - P_{\text{sprint}}V_{\max}\left(DELTA - TI_{C+1} - TO_{C+1} - K_0\right)$$

$$-\left(P_{C+1}D'_{2,C+1} - P_{\text{sprint}}V_{\max}\right)\left\{(LT_{C+1} + K_1)\sum_{k=1}^{9}\left(FR_{C+1,j,0} + FR_{C+1,j,4}\right)\right.$$

$$\left.+ RT_{C+1}\left[\sum_{j=1}^{9}\left(FR_{C+1,j,0} + FR_{C+1,j,-1}\right) + \sum_{j=8}^{9}\left(FR_{C,j,1} + FR_{C-1,j,2} + FR_{C-2,j,3}\right)\right]\right\}$$

- $$\sum_{i=1}^{C}\left\{P_i\left(D'_{2,i} - D'_{4,i}\right)\left[(LT_i + K_1)\sum_{j=1}^{7}\sum_{k=1}^{3} x_{i,j,k} + RT_i\sum_{j=1}^{7}\left(x_{i-1,j,1} + x_{i-2,j,2} + x_{i-3,j,3}\right)\right]\right\}$$

$$+\left(P_{C+1}D'_{2,C+1} - P_{\text{sprint}}V_{\min}\right)RT_{C+1}\sum_{j=1}^{7}\left(x_{C,j,1} + x_{C-1,j,2} + x_{C-2,j,3}\right) - S6$$

$$\le PIM + A - \sum_{i=1}^{C}\left\{P_i\left[D_{1,i} + D_{3,i} + D'_{4,i}(LC_i - TI_i - TO_i)\right.\right.$$

$$+\left(D'_{2,i} - D'_{4,i}\right)\left\{K_0 + (LT_i + K_1)\left[\sum_{j=1}^{9}\left(FR_{i,j,0} + FR_{i,j,4}\right) + \sum_{j=8}^{9}\sum_{k=1}^{3} FR_{i,j,k}\right]\right.$$

$$\left.\left.\left.+RT_i\left[\sum_{j=1}^{9}\left(FR_{i,j,0} + FR_{i,j,-1}\right) + \sum_{j=8}^{9}\left(FR_{i-1,j,1} + FR_{i-2,j,2} + FR_{i-3,j,3}\right)\right]\right\}\right]\right\}$$

$$- P_{C+1}\left(D_{1,C+1} + D_{3,C+1} + K_0D'_{2,C+1}\right) - P_{\text{sprint}}V_{\max}(DELTA - TI_{C+1} - TO_{C+1} - K_0)$$

$$-\left(P_{C+1}D'_{2,C+1} - P_{\text{sprint}}V_{\max}\right)\left\{(LT_{C+1} + K_1)\sum_{j=1}^{9}\left(FR_{C+1,j,0} + FR_{C+1,j,4}\right) + RT_{C+1}\right.$$

$$\left.\left[\sum_{j=1}^{9}\left(FR_{C+1,j,0} + FR_{C+1,j,-1}\right) + \sum_{j=8}^{9}\left(FR_{C,j,1} + FR_{C-1,j,2} + FR_{C-2,j,3}\right)\right]\right\}$$

$$\begin{array}{ll} x_{i,j,k} \ge FR_{i,j,k}, & \forall i,j,k \\ x_{i,j,k} \ge 0, & \forall i,j,k \\ S1_{i,j}, S2_{i,j}, S3_i, S4_j, S5, S6 \ge 0, & \forall i,j \end{array}$$

$x_{i,j,1} = 0$ for selected combinations of j and i.

$x_{i,j,2} = 0$ for selected combinations of j and i.

$x_{i,j,3} = 0$ for selected combinations of j and i.

$$x_{c,j,2} = 0, \quad \forall j.$$
$$x_{c,j,3} = 0, \quad \forall j.$$
$$x_{c-1,j,3} = 0, \quad \forall j.$$

Optional Constraint and bound:

- $$\sum_{l=1}^{i}\left\{P_l\left(D'_{2,l} - D'_{4,l}\right)\left[(LT_l + K_1)\sum_{j=1}^{7}\sum_{k=1}^{3} x_{l,j,k} + RT_l\sum_{j=1}^{7}\left(x_{l-1,j,1} + x_{l-2,j,2} + x_{l-3,j,3}\right)\right]\right\} - S7_i$$

$$< \sum_{l=1}^{i} EXPDIST_l - EPSON - \sum_{l=1}^{i}\left\{\left[\left[D_{1,l} + D_{3,l} + D'_{4,l}(LC_l - TI_l - TO_l) + \left(D'_{2,l} - D'_{4,l}\right)\right.\right.\right.$$

$$\left\{K_0 + (LT_l + K_l)\left[\sum_{j=1}^{9}\left(FR_{l,j,0} + FR_{l,j,4}\right) + \sum_{j=8}^{9}\sum_{k=1}^{3} FR_{l,j,k}\right]\right.$$

$$\left.\left.\left.\left. + RT_l\left[\sum_{j=1}^{9}\left(FR_{l,j,0} + FR_{l,j,-1}\right) + \sum_{j=8}^{9}\left(FR_{l-1,j,1} + FR_{l-2,j,2} + FR_{l-3,j,3}\right)\right]\right\}\right]\right]\right\}, \quad \forall i$$

- $S7_i \geq 0, \forall i$

III. MODEL OUTPUT

Results of the model will be the number of sorties, with respect to aircraft model, of different lengths, to schedule for each cycle. This could be one of many aircraft combinations, which based upon launch and recoveries, will provide the maximum number of sorties. When using these results to plan flight operations, the key is not how many aircraft of a specific model should be scheduled, but rather the total number and length of launches and total number of recoveries expected for a cycle. Only those sorties which are fixed requirements or meet specific airborne requirements must be scheduled according to aircraft model. All additional sorties can be modified for aircraft model as long as the air assets exist. For example, if cycle 1 scheduled seven F-14 single cycle sorties and only four were required, then the remaining three F-14 sorties could be substituted with single cycle sorties using other aircraft.

IV. SAMPLE SCENARIO

Appendix A provides a sample transit scenario with an associated flight operations period for scheduling optimization using COLA. Appendix B is the input data (Phase 1) containing the requirements and weather for the transit, along with the minimum defensive posture to be included during flight operations. Since this is only an unclassified example, actual launch and recovery times and turning rates are not utilized. The data used is only for purposes of showing the capabilities of the model. A listing of the GAMS model constructed from the input is provided in Appendix C (Phase 2). The corresponding results from execution of the model using both Relaxed Mixed Integer Programming on a personal computer, and Mixed Integer Programming on a mainframe system are provided in Appendices D and E, respectively (Phase 3).

The solution of the model using both methods of optimization is summarized in Table 1 and 2. The maximum number of sorties to launch per cycle is compared in Table 1 and shows a maximum deviation of five with both techniques resulting in a higher number of sorties to be launched in the four cycle flight operations period. This number is much higher than would actually be encountered, but nevertheless, it implies that the environmental conditions used for the launch and recovery of aircraft is not a significant factor in impeding the CV from meeting its transit requirements. When an actual flight operations period is being planned the use of more definitive requirements, true launch and recovery times, and accurate carrier maneuvering parameters would probably decrease the optimal number of sorties. The discrepancy between the two solutions is a result of rounding up the decision variables with real results in the

RMIP to the next highest integer to ensure an aircraft is scheduled to meet a specifically tasked mission or an additionally tasked sortie.

TABLE 1

	RMIP/MIP		
	Single Cycle	**Double Cycle**	**Triple Cycle**
Cycle 1	24/23	1/2	0/0
Cycle 2	19/19	3/4	0/0
Cycle 3	16/16	8/6	0/0
Cycle 4	25/20	0/0	0/0
Total by length	84/78	11/12	0/0

Total sorties during flight operations period: 95/90

Table 2, which arranges the output data by the different combinations of aircraft type and sortie length for each cycle shows there is a difference in the selection of combinations by the two different models in only 14 of a possible 44 combinations. Of these 14, only three differ by more than one sortie.

TABLE 2. SINGLE CYCLE/DOUBLE CYCLE

		F14	F18	A6	EA6	E2	S3	ES3
Cycle 1	RMIP	6/NA	8/NA	5/0	1.0	1/1	2/0	1/NA
	MIP	6/NA	8/NA	4.0	1.0	0/2	3/0	1/NA
Cycle 2	RMIP	6/NA	8/NA	2/2	1/0	1/0	0/1	1/NA
	MIP	6/NA	8/NA	4/2	0/1	0/1	0/1	1/NA
Cycle 3	RMIP	6/NA	8/NA	0/4	0/1	0/1	1/2	1/NA
	MIP	6/NA	8/NA	0/3	0/1	0/1	1/1	1/NA
Cycle 4	RMIP	6/NA	8/NA	4/0	3/0	3/0	0/0	1/NA
	MIP	4/NA	8/NA	4/0	0/0	2/0	1/0	1/NA

This variation between the two models is minimal, and because of influences which cannot be predicted, the use of a RMIP model instead of a MIP is a viable alternative to provide a relatively quick solution on available ship board systems.

Finally, a comparison of the estimated position of the CV after each cycle generated as a result of both optimization models shows the positions to be almost exact with the greatest deviation for a cycle position being 3 NM. This small difference also supports the use of the RMIP model instead of the MIP model.

The COLA model can be easily modified to allow a Battle Group Commander to determine the minimum number of sorties to launch to meet the tactical requirements. This is accomplished by changing the objective from a maximization problem to a minimization problem, and eliminating the constraints associated with the CV's distance of travel along the PIM. Appendix F provides the result of this model using RMIP on a personal computer.

Another constraint which is not presently incorporated into the model could be the requirement to launch a minimum number of sorties for each aircraft type. This would be:

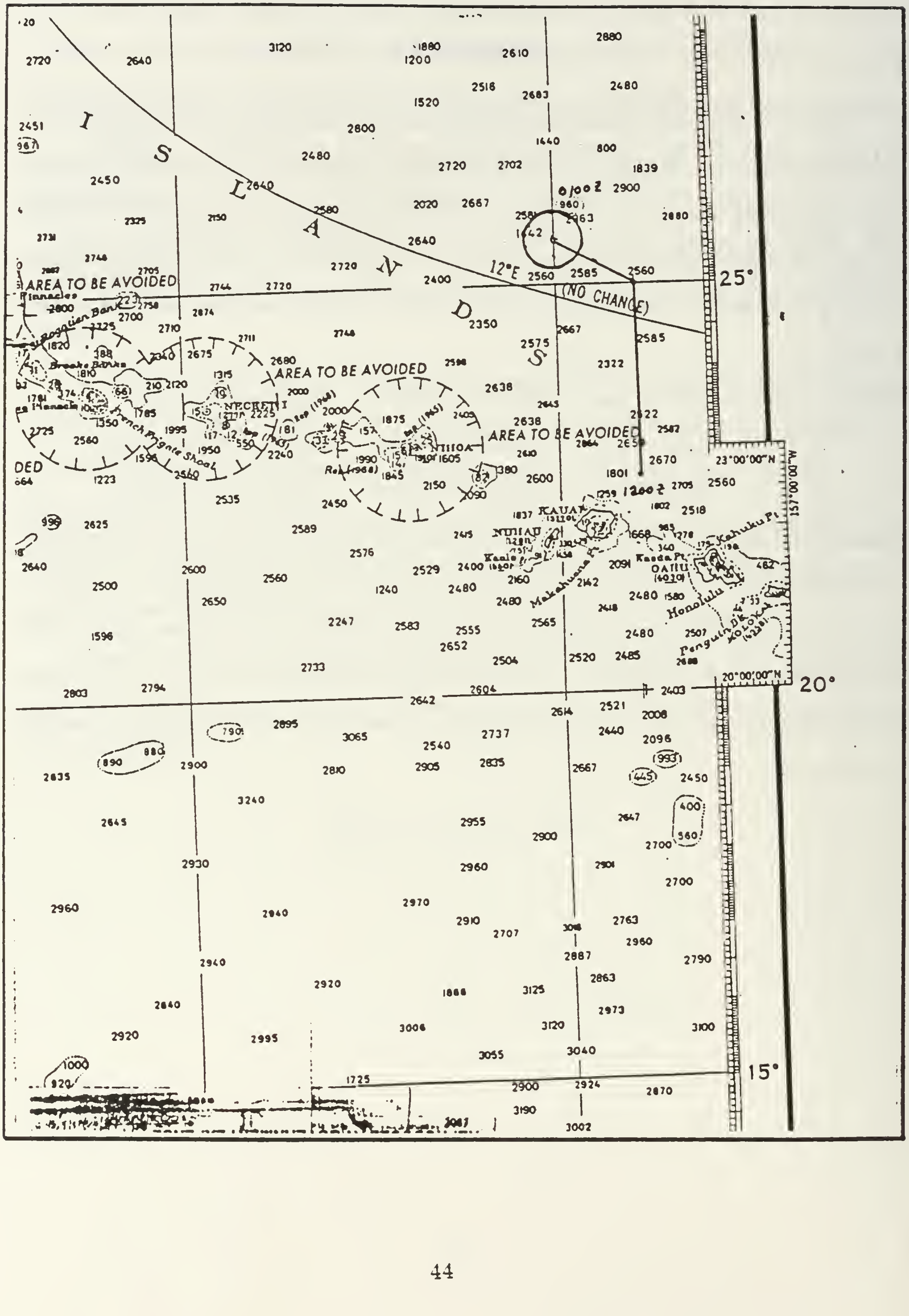
$$\sum_{i=1}^{c}\sum_{k=1}^{3} x_{i,j,k} \geq \min_j, \ \ \forall_j.$$

APPENDIX A

Scenario: A carrier battle group is departing Pearl Harbor, Hawaii, en route to the Barents Sea. It will begin its first flight operations period at 1200, 30 July 1990, consisting of four cycle, each of one hour, 45 minutes in duration. At 0100, 31 July 1990, the CVBG must pass through a circular area of 20 NM radius which is being sanitized to determine if a Soviet SSN has been assigned to shadow the CVBG.

PIM of concern:	1200 30 July	25-40 N 159-05W	flight ops begins
	2200 30 July	25-00N 159-10W	course/speed change
	0100 31 July	25-30N 160-00W	sanitation rendezvous

Specific tasking, weather and CV speed requirements are provided in the COLP worksheet. The CVBG commander is not concerned with position at the end of each cycle, only with meeting the sanitation requirement. Relaxation of any of the constraints is not authorized. The maximum additional sorties to launch per cycle of any aircraft type is 4. The maximum number of tactical sorties to launch per cycle is 25.


I S L A N D S
AREA TO BE AVOIDED
AREA TO BE AVOIDED
AREA TO BE AVOIDED
12°E
(NO CHANGE)
0100Z
1200Z
25°
20°
15°
23°00'00"N
20°00'00"N
157°00'00"W
French Frigate Shoal
NECKER I
NIHOA
KAUAI
NIIHAU
Kaula
OAHU
Honolulu
Kahuku Pt
Penguin Bk
MOLOKAI

APPENDIX B

CARRIER OPTIMIZATION LAUNCH PROGRAM WORKSHEET

Part I: Generic Information

A. DATE and TIME flight operations begin: Date 90211 Time _1200_

LATITUDE/LONGITUDE at begin of flt ops: Lat ___22-40N_________
Long __159-05W_________

B. DATE and TIME of next flight operations
period or rendezvous point: Date 90212 Time _0100_

LATITUDE/LONGITUDE at end/rendezvous: Lat ___25-30N_________
Long __160-00W_________

C. PIM Position Points between positions given in A and B.

	Date	Time	Latitude	Longitude
1.	_90211__	__2200__	___25-00N___	__159-10W___
2.	________	________	____________	____________
3.	________	________	____________	____________
4.	________	________	____________	____________
5.	________	________	____________	____________
6.	________	________	____________	____________
7.	________	________	____________	____________
8.	________	________	____________	____________

D. MAXIMUM SPRINT VELOCITY WHEN NOT IN FLT OPS: _20_

E. DATE and TIME Flight Operations period starting in A ends:

Date _90211__ Time __1900__

Number of Cycles: _4_ All same length

If same length, length is: _105__

If different lengths, lengths are:

1. _____	5. _____	9. _____	13. _____	17. _____
2. _____	6. _____	10. _____	14. _____	18. _____
3. _____	7. _____	11. _____	15. _____	19. _____
4. _____	8. _____	12. _____	16. _____	20. _____

F. MAXIMUM VELOCITY BETWEEN CYCLES: _18__

G. MAXIMUM VELOCITY ON TURNS: _15__

CARRIER OPTIMIZATION LAUNCH PROGRAM WORKSHEET

Part I: Generic Information

H. Distances allowed to deviate from PIM at rendezvous point:

BEHIND __20__ AHEAD __10__

I. WEATHER Information:

Cycle	Time	W/V	SS	Vis	Ceil	Cycle	Time	W/V	SS	Vis	Ceil
1	1200	040/10	_2	_10	_5k_	11		/			
2	1345	045/10	_2	_10	4.5k	12		/			
3	1530	060/15	_3	__7	_3k_	13		/			
4	1715	070/10	_2	__5	_4k_	14		/			
5	1900	090/05	_1	__5	_4k_	15		/			
6		/				16		/			
7		/				17		/			
8		/				18		/			
9		/				19		/			
10		/				20		/			
						21		/			

J. SUNRISE / SUNSET Data

Date	Sunrise	Sunset
_90211	__0500_	_2030_
_90212	__0501_	_2029_

CARRIER OPTIMIZATION LAUNCH PROGRAM WORKSHEET

Part II: Flight Analyzer Data

Cycle	Single Cycle Launches	Double Cycle Launches	Triple Cycle Launches	Yo-Yo Cycle Launches	Returning Cycle Launches	Originating Cycle Launches
1	____	____	____	____	____	____
2	____	____	____	____	____	____
3	____	____	____	____	____	____
4	____	____	____	____	____	____
5	____	____	____	____	____	____
6	____	____	____	____	____	____
7	____	____	____	____	____	____
8	____	____	____	____	____	____
9	____	____	____	____	____	____
10	____	____	____	____	____	____
11	____	____	____	____	____	____
12	____	____	____	____	____	____
13	____	____	____	____	____	____
14	____	____	____	____	____	____
15	____	____	____	____	____	____
16	____	____	____	____	____	____
17	____	____	____	____	____	____
18	____	____	____	____	____	____
19	____	____	____	____	____	____
20	____	____	____	____	____	____
21	____	____	____	____	____	____

CARRIER OPTIMIZATION LAUNCH PROGRAM WORKSHEET

Part III: Flight Optimization (COLA) Data

A. AIRCRAFT AVAILABLE (A > D + E + F)

F14	F18	A6	EA6	E2	S3	ES3	TKR	COD	SH3
_20	_18	_10	_4_	__4	__4	_2_	_2_	_0_	_4_

B. MINIMUM HOURS REQUIRED BY MODEL

F14	F18	A6	EA6	E2	S3	ES3	TKR	COD	SH3
_8_	_8_	_6_	_6_	_8_	_8_	_4_	_0_	_0_	_0_

C. MAXIMUM CYCLE LENGTH BY MODEL

F14	F18	A6	EA6	E2	S3	ES3	TKR	COD	SH3
_1_	_1_	_2_	_2_	_2_	_2_	_1_	_1_	_0_	_0_

D. FIXED TACTICAL SORTIES BY MODEL AND CYCLE LENGTH
(single cycle / double cycle / triple cycle)

	F14	F18	A6	EA6	E2	S3	ES3	TKR	COD	SH3
1	_/_/_	_/_/_	_/_/_	_/_/_	_/1/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_
2	_/_/_	_/_/_	_/2/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_
3	_/_/_	_/_/_	_/1/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_
4	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_
5	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_
6	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_
7	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_
8	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_
9	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_
10	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_
11	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_
12	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_
13	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_
14	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_
15	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_
16	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_
17	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_
18	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_
19	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_
20	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_
21	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_	_/_/_

CARRIER OPTIMIZATION LAUNCH PROGRAM WORKSHEET

Part III: Flight Optimization Data

E. AIRCRAFT OTHER THAN D & F REQUIRED AIRBORNE BY TYPE AND CYCLE

	F14	F18	A6	EA6	E2	S3	ES3	TKR	COD	SH3
1	2	4	2	1	1	2	1	0	0	2
2	2	4	2	1	1	1	1	0	0	2
3	2	4	2	1	1	1	1	0	0	2
4	2	4	2	1	1	2	1	0	0	2
5										
6										
7										
8										
9										
10										
11										
12										
13										
14										
15										
16										
17										
18										
19										
20										
21										

CARRIER OPTIMIZATION LAUNCH PROGRAM WORKSHEET

Part III: Flight Optimization Data

F. FIXED LOGICAL SORTIES BY MODEL
(non-originating / non-returning / Yo-Yo)

	F14	F18	A6	EA6	E2	S3	ES3	TKR	COD	SH3
1	__/__	__/__	__/__	__/__	__/__	__/__	__/__	_/_/1	__/__	_/_/_
2	__/__	__/__	__/__	__/__	__/__	__/__	__/__	_/_/1	__/__	_/_/_
3	__/__	__/__	__/__	__/__	__/__	__/__	__/__	_/_/1	__/__	_/_/_
4	__/__	__/__	__/__	__/__	__/__	__/__	__/__	_/_/1	__/__	_/_/_
5	__/__	__/__	__/__	__/__	__/__	__/__	__/__	_/_/1	__/__	_/_/_
6	__/__	__/__	__/__	__/__	__/__	__/__	__/__	_/_/_	__/__	_/_/_
7	__/__	__/__	__/__	__/__	__/__	__/__	__/__	_/_/_	__/__	_/_/_
8	__/__	__/__	__/__	__/__	__/__	__/__	__/__	_/_/_	__/__	_/_/_
9	__/__	__/__	__/__	__/__	__/__	__/__	__/__	_/_/_	__/__	_/_/_
10	__/__	__/__	__/__	__/__	__/__	__/__	__/__	_/_/_	__/__	_/_/_
11	__/__	__/__	__/__	__/__	__/__	__/__	__/__	_/_/_	__/__	_/_/_
12	__/__	__/__	__/__	__/__	__/__	__/__	__/__	_/_/_	__/__	_/_/_
13	__/__	__/__	__/__	__/__	__/__	__/__	__/__	_/_/_	__/__	_/_/_
14	__/__	__/__	__/__	__/__	__/__	__/__	__/__	_/_/_	__/__	_/_/_
15	__/__	__/__	__/__	__/__	__/__	__/__	__/__	_/_/_	__/__	_/_/_
16	__/__	__/__	__/__	__/__	__/__	__/__	__/__	_/_/_	__/__	_/_/_
17	__/__	__/__	__/__	__/__	__/__	__/__	__/__	_/_/_	__/__	_/_/_
18	__/__	__/__	__/__	__/__	__/__	__/__	__/__	_/_/_	__/__	_/_/_
19	__/__	__/__	__/__	__/__	__/__	__/__	__/__	_/_/_	__/__	_/_/_
20	__/__	__/__	__/__	__/__	__/__	__/__	__/__	_/_/_	__/__	_/_/_
21	__/__	__/__	__/__	__/__	__/__	__/__	__/__	_/_/_	__/__	_/_/_

G. MAXIMUM NON-PLANNED SORTIES BY MODEL (default 5): __5__

H. MAXIMUM COMBINED SORTIES ALLOWED PER CYCLE (default 25): _25__

I. DISTANCE ALLOWED BEHIND AT END OF EACH CYCLE: _N/A__

APPENDIX C

```
$OFFUPPER
SETS
   I  all possible cycles /1* 5/
   E(I)  entire length cycles /1* 4/
   J  all aircraft type /F14,F18,A6,EA6,E2,S3,ES3,TKR,COD/
   O(J)  aircraft of concern /F14,F18,A6,EA6,E2,S3,ES3/
   N(J)  other aircraft  /TKR,COD/
   K  possible length of flight in cycles /NS,L0,L1,L2,L3,NR/
   M(K)  length of tactical flight in cycles /L1,L2,L3/
   A(K)  special launches /L0,NR/
   B(K)  special recoveries /NS,L0/;
ALIAS (E,EP);
PARAMETERS
   LC(E)  length of cycle E in minutes
         / 1   105
           2   105
           3   105
           4   105  /
   LR(I)   time in minutes required to launch one aircraft in cycle
           I
         / 1    1.09300
           2    1.09300
           3    1.09300
           4    1.09300
           5    1.09300  /
   RR(I)   time in minutes required to recover one aircraft in
           cycle I
         / 1    1.73300
           2    1.73300
           3    1.73300
           4    1.73300
           5    1.73300  /
   TI(I)  time in minutes to turn into the wind for cycle I
         / 1     5.84398
           2     6.78663
           3     9.54318
           4    11.74558
           5     0.00000  /
   TO(I)  time in minutes to turn back to PIM after cycle I
         / 1     6.08849
           2     7.44878
           3    10.34931
           4    13.51245
           5     0.01280  /
   H(O)   minutes required to be flown by aircraft of type O
         / F14    480
           F18    480
           A6     360
           EA6    360
```

```
            E2     480
            S3     480
            ES3    240  /
    D1(I)  distance along track phase 1 of cycle I
         /  1       1.33448
            2       1.50035
            3       1.85888
            4       1.98948
            5       0.00000  /
    D2(I)  multiple of distance along track phase 2 of cycle I
         /  1       0.18621
            2       0.16531
            3       0.06149
            4       0.02560
            5       0.45421  /
    D3(I)  distance along track phase 3 of cycle I
         /  1       1.39560
            2       1.66570
            3       2.06008
            4       2.42762
            5       0.00320  /
    D4(E)  multiple of distance along track phase 4 of cycle E
         /  1       0.29986
            2       0.29897
            3       0.29848
            4       0.29271  /
    P(I)   correction to determine approx Pim distance of cycle I
         /  1    1.00000
            2    0.99953
            3    0.99358
            4    0.97717
            5    0.90689  /;
PARAMETER W(E) sum of D1 and D3 and D4 times travel time for each
          cycle;
          W(E) = D1(E) + D3(E) + D4(E) * (LC(E) - TI(E) - TO(E));
PARAMETER Y(E) difference between D2 and D4 for each cycle;
          Y(E) = D2(E) - D4(E) ;
PARAMETER XCOPY(E,O,M) rounded copy of x ;
PARAMETER V1 infeasibility flag 1 ;
PARAMETER V2 infeasibility flag 2 ;
PARAMETER V3 infeasibility flag 3 ;
PARAMETER V4 infeasibility flag 4 ;
PARAMETER V5 infeasibility flag 5 ;
TABLE FR(I,J,K)  specific tasked number of aircraft
        F14.NS F14.L0 F14.L1 F14.L2 F14.L3 F14.NR
      1      0      0      0      0      0      0
      2      0      0      0      0      0      0
      3      0      0      0      0      0      0
      4      0      0      0      0      0      0
      5      0      0      0      0      0      0
   +    F18.NS F18.L0 F18.L1 F18.L2 F18.L3 F18.NR
      1      0      0      0      0      0      0
      2      0      0      0      0      0      0
      3      0      0      0      0      0      0
      4      0      0      0      0      0      0
```

```
    5       0       0       0       0       0       0
+       A6.NS   A6.L0   A6.L1   A6.L2   A6.L3   A6.NR
    1       0       0       0       0       0       0
    2       0       0       0       2       0       0
    3       0       0       0       1       0       0
    4       0       0       0       0       0       0
    5       0       0       0       0       0       0
+      EA6.NS  EA6.L0  EA6.L1  EA6.L2  EA6.L3  EA6.NR
    1       0       0       0       0       0       0
    2       0       0       0       0       0       0
    3       0       0       0       0       0       0
    4       0       0       0       0       0       0
    5       0       0       0       0       0       0
+       E2.NS   E2.L0   E2.L1   E2.L2   E2.L3   E2.NR
    1       0       0       0       1       0       0
    2       0       0       0       0       0       0
    3       0       0       0       0       0       0
    4       0       0       0       0       0       0
    5       0       0       0       0       0       0
+       S3.NS   S3.L0   S3.L1   S3.L2   S3.L3   S3.NR
    1       0       0       0       0       0       0
    2       0       0       0       0       0       0
    3       0       0       0       0       0       0
    4       0       0       0       0       0       0
    5       0       0       0       0       0       0
+      ES3.NS  ES3.L0  ES3.L1  ES3.L2  ES3.L3  ES3.NR
    1       0       0       0       0       0       0
    2       0       0       0       0       0       0
    3       0       0       0       0       0       0
    4       0       0       0       0       0       0
    5       0       0       0       0       0       0
+      TKR.NS  TKR.L0  TKR.L1  TKR.L2  TKR.L3  TKR.NR
    1       0       1       0       0       0       0
    2       0       1       0       0       0       0
    3       0       1       0       0       0       0
    4       0       1       0       0       0       0
    5       0       1       0       0       0       0
+      COD.NS  COD.L0  COD.L1  COD.L2  COD.L3  COD.NR
    1       0       0       0       0       0       0
    2       0       0       0       0       0       0
    3       0       0       0       0       0       0
    4       0       0       0       0       0       0
    5       0       0       0       0       0       0 ;
TABLE OR(E,O) minimum additional number of aircraft
       F14   F18    A6   EA6    E2    S3   ES3
   1     2     4     2     1     1     2     1
   2     2     4     2     1     1     1     1
   3     2     4     2     1     1     1     1
   4     2     4     2     1     1     2     1 ;
TABLE AC(E,O)  maximum aircraft of type T available during cycle C
       F14   F18    A6   EA6    E2    S3   ES3
   1    20    18    10     4     4     4     2
   2    20    18    10     4     4     4     2
   3    20    18    10     4     4     4     2
   4    20    18    10     4     4     4     2 ;
```

```
TABLE MS(E,O) maximum sorties per cycle E of aircraft O
       F14  F18   A6  EA6   E2    S3  ES3
    1    6    8    6    5    5    6    5
    2    6    8    6    5    5    5    5
    3    6    8    6    5    5    5    5
    4    6    8    6    5    5    6    5 ;
SCALARS  DELT                    /    360.00000 /
         PLAST                   /   0.90621 /
         AHEAD                   /  10 /
         BEHIND                  /  20 /
         VMAX                    /  20 /
         VMIN                    /   5 /
         PIM                     /    194.34317 /;
VARIABLES
          X(E,O,M)  number of aircraft to launch
          Z         total number of sorties
          S1(E,O)   elastic variable
          S2(E,O)   elastic variable
          S3(E)     elastic variable
          S4(O)     elastic variable
          S5        elastic variable
          S6        elastic variable;
INTEGER VARIABLE X;
POSITIVE VARIABLE S1(E,O);
POSITIVE VARIABLE S2(E,O);
POSITIVE VARIABLE S3(E);
POSITIVE VARIABLE S4(O);
POSITIVE VARIABLE S5;
POSITIVE VARIABLE S6;
   X.LO(E,O,M) = FR(E,O,M) ;
   X.UP(E,O,M) = MIN(MS(E,O),AC(E,O)) ;
   S1.FX(E,O)  = 0 ;
   S5.FX       = 0 ;
   S6.FX       = 0 ;
   X.UP('4',O,'L2') = 0 ;
   X.UP('4',O,'L3') = 0 ;
   X.UP('3',O,'L3') = 0 ;
   X.UP(E,'F14','L2') = 0 ;
   X.UP(E,'F18','L2') = 0 ;
   X.UP(E,'ES3','L2') = 0 ;
   X.UP(E,'F14','L3') = 0 ;
   X.UP(E,'F18','L3') = 0 ;
   X.UP(E,'A6 ','L3') = 0 ;
   X.UP(E,'EA6','L3') = 0 ;
   X.UP(E,'E2 ','L3') = 0 ;
   X.UP(E,'S3 ','L3') = 0 ;
   X.UP(E,'ES3','L3') = 0 ;
EQUATIONS
     SORTIES   total number of sorties to fly
     MINAIR(E,O)       minimum aircraft required airborne
     MAXSORTIE(E,O)    maximum sorties of aircraft type allowed
     CYCLESORT(E)  maximum sorties per cycle
     MAXAVAIL(E,O)     maximum available aircraft
     FLIGHTHR(O)       flight hour requirements
     CYCLETIME(E)      cycle time available
```

```
     DIST1            min dist travelled relative to PIM
     DIST2            max dist travelled relative to PIM ;
SORTIES .. Z =E= SUM((E,O,M),X(E,O,M)) - SUM((E,O),10*S1(E,O) +
                 10*S2(E,O)) - SUM(E,10*S3(E)) - SUM(O,10*S4(O)) -
                 10*S5 - 15*S6;
MINAIR(E,O) .. SUM(M,X(E,O,M)) + X(E-1,O,'L2')
               +X(E-2,O,'L3')+S1(E,O) =G=OR(E,O)+SUM((M),FR(E,O,M))
               + FR(E-1,O,'L2') + FR(E-1,O,'L3') + FR(E-2,O,'L3');
MAXSORTIE(E,O) .. SUM(M,X(E,O,M)) - S2(E,O) =L= MS(E,O) ;
CYCLESORT(E) .. SUM((O,M),X(E,O,M)) - S3(E) =L= 25 ;
MAXAVAIL(E,O) .. SUM(M,X(E,O,M)) + SUM(M,X(E-1,O,M)) +X(E-2,O,'L2')
                 + X(E-2,O,'L3') + X(E-3,O,'L3') =L= AC(E,O)
                 - FR(E,O,'L0') - SUM(EP$(ORD(EP) LE ORD(E)),
                 FR(EP,O,'NR'));
FLIGHTHR(O)..SUM(E,(LC(E)*X(E,O,'L1'))+(LC(E)+LC(E-1))
             *X(E-1,O,'L2')+(LC(E)+LC(E-1)+LC(E-2))
             *X(E-2,O,'L3')) + S4(O) =G= H(O);
CYCLETIME(E) .. (LR(E)-0.145)*SUM((O,M),X(E,O,M)) + (0.9375+RR(E))*
                SUM(O,(X(E-1,O,'L1')+X(E-2,O,'L2')+X(E-3,O,'L3')))
                =L=LC(E) - TI(E) -TO(E) - 3.62 - (LR(E)-0.145)
                *(SUM((T,A),FR(E,T,A))+SUM((N,M),FR(E,N,M)))
                -(0.9375+RR(E))*(SUM((T,B),FR(E,T,B))
                +SUM(N,(FR(E-1,N,'L1') + FR(E-2,N,'L2')
                + FR(E-3,N,'L3'))));
DIST1 .. RR('5')*(P('5')*D2('5')-VMIN*PSPRINT/60)
         *SUM(O,(X('4',O,'L1')+X('3',O,'L2')+X('2',O,'L3')))
         +SUM(E,(P(E)*Y(E)*((LR(E)-0.145)*SUM((O,M),X(E,O,M))
         +RR(E)*SUM(O,(X(E-1,O,'L1')+X(E-2,O,'L2')+
         X(E-3,O,'L3'))))))-S5 =L=PIM+AHEAD-VMIN*(DELT-TO('5')
         -TI('5'))*PSPRINT/60 - P('5')*(D1('5')+D3('5'))
         -(P('5')*D2('5')-VMIN*PSPRINT/60)*(LR('5')-0.145)
         *SUM((J,A),FR('5',J,A))-RR('5')*(P('5')*D2('5')
         -VMIN*PSPRINT/60)*(SUM((J,B),FR('5',J,B))+
         SUM(N,(FR('4',N,'L1')+FR('3',N,'L2')+FR('2',N,'L3'))))
         -3.62*(P('5')*D2('5')-VMIN*PSPRINT/60)-SUM(E,(P(E)*W(E)
         +P(E)*Y(E)*(LR(E)-0.145)*(SUM((J,A),FR(E,J,A))
         +SUM((N,M),FR(E,N,M)))+P(E)*Y(E)*RR(E)
         *(SUM((J,B),FR(E,J,B))+SUM(N,(FR(E-1,N,'L1')+
         FR(E-2,N,'L2')+FR(E-3,N,'L3'))))+P(E)*Y(E)*3.62))-P('5')
         *D3('5');
DIST2 .. RR('5')*(P('5')*D2('5')-VMAX*PSPRINT/60)
         *SUM(O,(X('4',O,'L1')+X('3',O,'L2')+X('2',O,'L3')))
         +SUM(E,(P(E)*Y(E)*((LR(E)-0.145)*SUM((O,M),X(E,O,M))
         +RR(E)*SUM(O,(X(E-1,O,'L1')+X(E-2,O,'L2')+
         X(E-3,O,'L3'))))))-S5 =G= PIM-BEHIND-VMAX*(DELT-TO('5')
         -TI('5'))*PSPRINT/60 - P('5')*(D1('5')+D3('5'))
         -(P('5')*D2('5')-VMAX*PSPRINT/60)*(LR('5')-0.145)
         *SUM((J,A),FR('5',J,A))-RR('5')*(P('5')*D2('5')
         -VMAX*PSPRINT/60)*(SUM((J,B),FR('5',J,B))+
         SUM(N,(FR('4',N,'L1')+FR('3',N,'L2')+FR('2',N,'L3'))))
         -3.62*(P('5')*D2('5')-VMAX*PSPRINT/60)-SUM(E,(P(E)*W(E)
         +P(E)*Y(E)*(LR(E)-0.145)*(SUM((J,A),FR(E,J,A))
         +SUM((N,M),FR(E,N,M)))+P(E)*Y(E)*RR(E)
         *(SUM((J,B),FR(E,J,B))+SUM(N,(FR(E-1,N,'L1')+
         FR(E-2,N,'L2')+FR(E-3,N,'L3'))))+P(E)*Y(E)*3.62))-P('5')
```

```
        *D3('5');



**************** FOR RELAXED INTEGER PROGRAM *****************

MODEL COLP /ALL/ ;
OPTIONS OPTCR = 0.05 , DECIMALS = 1 , ITERLIM = 10000 ;
OPTIONS LIMROW = 0 , LIMCOL = 0 , SYSOUT = OFF , SOLPRINT = OFF ;
SOLVE COLP USING RMIP MAXIMIZING Z ;
V1 = SUM((E,O),S1.L(E,O)) ;
V2 = SUM((E,O),S2.L(E,O)) ;
V3 = SUM(E,S3.L(E)) ;
V4 = SUM(O,S4.L(O)) ;
V5 = S5.L + S6.L ;
DISPLAY V1,V2,V3,V4,V5;
X.L(E,O,M) = ROUND(X.L(E,O,M)) ;
OPTION X:0:1:2;
DISPLAY X.L;



****************  FOR INTEGER PROGRAMMING *******************

MODEL COLP /ALL/ ;
OPTIONS OPTCR = 0.00 , DECIMALS = 1 , ITERLIM = 100000 , WORK =
9999 ;
OPTIONS LIMROW = 0 , LIMCOL = 0 , SYSOUT = OFF , SOLPRINT = OFF ;
SOLVE COLP USING MIP MAXIMIZING Z ;
V1 = SUM((E,O),S1.L(E,O)) ;
V2 = SUM((E,O),S2.L(E,O)) ;
V3 = SUM(E,S3.L(E)) ;
V4 = SUM(O,S4.L(O)) ;
V5 = S5.L + S6.L ;
DISPLAY V1,V2,V3,V4,V5;
OPTION X:0:1:2;
DISPLAY X.L;
```

APPENDIX D

GENERAL ALGEBRAIC MODELING SYSTEM

```
MODEL STATISTICS    SOLVE COLP USING RMIP FROM LINE 258

               S O L V E      S U M M A R Y

     MODEL   COLA                 OBJECTIVE  Z
     TYPE    RMIP                 DIRECTION  MAXIMIZE
     SOLVER  ZOOM                 FROM LINE  258

**** SOLVER STATUS     1 NORMAL COMPLETION
**** MODEL STATUS      1 OPTIMAL
**** OBJECTIVE VALUE               96.0788

**** REPORT SUMMARY :        0     NONOPT
                             0 INFEASIBLE
                             0  UNBOUNDED

----     264 PARAMETER V1         =        0.0 INFEASIBILITY FLAG 1
             PARAMETER V2         =        0.0 INFEASIBILITY FLAG 2
             PARAMETER V3         =        0.0 INFEASIBILITY FLAG 3
             PARAMETER V4         =        0.0 INFEASIBILITY FLAG 4
             PARAMETER V5         =        0.0 INFEASIBILITY FLAG 5

----     267 VARIABLE  X.L    NUMBER OF AIRCRAFT OF TYPE O TO LAUNCH
                              IN CYCLE E ON A LENGTH OF M CYCLES
```

	F14.L1	F18.L1	A6.L1	A6.L2	EA6.L1	EA6.L2
1	6	8	5		1	
2	6	8	2	2	1	
3	6	8		4		1
4	6	8	4		3	

+	E2.L1	E2.L2	S3.L1	S3.L2	ES3.L1
1	1	1	2		1
2	1			1	1
3		1	1	2	1
4	3				1

30 JUL 1990

CYCLE	START TIME	STOP TIME	AIRCRAFT LAUNCH	AIRCRAFT RECOVER	ESTIMATED POSIT AT END OF CYCLE		PIM POSIT AT END OF CYCLE		BEARING/DIST EP to PIM
1	1200	1345	26	1	23-07N	159-00W	23-04N	159-05W	241T/ 6NM
2	1345	1530	23	25	23-29N	158-47W	23-29N	159-06W	270T/ 18NM
3	1530	1715	25	21	23-44N	158-37W	23-53N	159-07W	288T/ 29NM
4	1715	1900	26	20	23-59N	158-22W	24-18N	159-08W	294T/ 46NM
Final	1900	2004	1	34	24-23N	158-40W	24-32N	159-08W	291T/ 27NM
Next	90212	0100			25-29N	159-59W	25-30N	160-00W	304T/ 0NM

30 JUL 1990

CYCLE 1 START TIME: 1200 STOPTIME: 1345

Scheduled Launches: 26 Scheduled Recoveries: 1

A/C TYPE	SINGLE CYCLE	DOUBLE CYCLE	TRIPLE CYCLE	YO-YO	NON ORIG	NON RET
F14	6	0	0	0	0	0
F18	8	0	0	0	0	0
A6	5	0	0	0	0	0
EA6	1	0	0	0	0	0
E2	1	1	0	0	0	0
S3	2	0	0	0	0	0
ES3	1	0	0	0	0	0
TKR	0	0	0	1	0	0
COD	0	0	0	0	0	0
SH3	0	0	0	0	0	0

30 JUL 1990

CYCLE 2 START TIME: 1345 STOPTIME: 1530

Scheduled Launches: 23 Scheduled Recoveries: 25

A/C TYPE	SINGLE CYCLE	DOUBLE CYCLE	TRIPLE CYCLE	YO-YO	NON ORIG	NON RET
F14	6	0	0	0	0	0
F18	8	0	0	0	0	0
A6	2	2	0	0	0	0
EA6	1	0	0	0	0	0
E2	1	0	0	0	0	0
S3	0	1	0	0	0	0
ES3	1	0	0	0	0	0
TKR	0	0	0	1	0	0
COD	0	0	0	0	0	0
SH3	0	0	0	0	0	0

30 JUL 1990

CYCLE 3 START TIME: 1530 STOPTIME: 1715

Scheduled Launches: 25 Scheduled Recoveries: 21

A/C TYPE	SINGLE CYCLE	DOUBLE CYCLE	TRIPLE CYCLE	YO-YO	NON ORIG	NON RET
F14	6	0	0	0	0	0
F18	8	0	0	0	0	0
A6	0	4	0	0	0	0
EA6	0	1	0	0	0	0
E2	0	1	0	0	0	0
S3	1	2	0	0	0	0
ES3	1	0	0	0	0	0
TKR	0	0	0	1	0	0
COD	0	0	0	0	0	0
SH3	0	0	0	0	0	0

30 JUL 1990

CYCLE 4 START TIME: 1715 STOPTIME: 1900

Scheduled Launches: 26 Scheduled Recoveries: 20

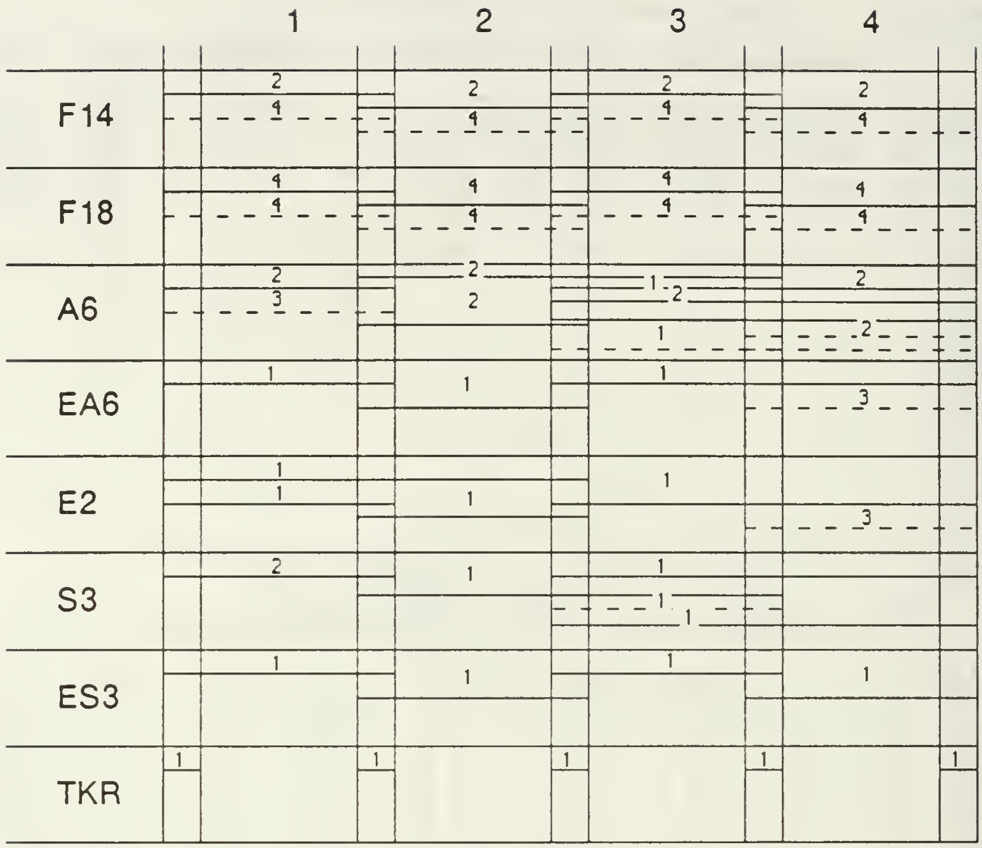
A/C TYPE	SINGLE CYCLE	DOUBLE CYCLE	TRIPLE CYCLE	YO-YO	NON ORIG	NON RET
F14	6	0	0	0	0	0
F18	8	0	0	0	0	0
A6	4	0	0	0	0	0
EA6	3	0	0	0	0	0
E2	3	0	0	0	0	0
S3	0	0	0	0	0	0
ES3	1	0	0	0	0	0
TKR	0	0	0	1	0	0
COD	0	0	0	0	0	0
SH3	0	0	0	0	0	0

30 JUL 1990

LAST RECOVERY: START TIME: 1900 STOPTIME: 2004

Scheduled Launches: 1 Scheduled Recoveries: 34

A/C TYPE	SINGLE CYCLE	DOUBLE CYCLE	TRIPLE CYCLE	YO-YO	NON ORIG	NON RET
F14	0	0	0	0	0	0
F18	0	0	0	0	0	0
A6	0	0	0	0	0	0
EA6	0	0	0	0	0	0
E2	0	0	0	0	0	0
S3	0	0	0	0	0	0
ES3	0	0	0	0	0	0
TKR	0	0	0	1	0	0
COD	0	0	0	0	0	0
SH3	0	0	0	0	0	0



——— Required airborne or specific tasked sorties

- - - - Optional sorties. If other aircraft are available and there is a preference to launch them, they can substitute for the optional sorties.

	Extra Sorties	
Cycle	Single	Double
1	11	0
2	8	0
3	9	1
4	16	0

Figure. Using RMIP on PC

APPENDIX E

GENERAL ALGEBRAIC MODELING SYSTEM

MODEL STATISTICS   SOLVE COLP USING MIP FROM LINE 258

```
              S O L V E     S U M M A R Y

     MODEL   COLA                 OBJECTIVE  Z
     TYPE    MIP                  DIRECTION  MAXIMIZE
     SOLVER  ZOOM                 FROM LINE  258

**** SOLVER STATUS     1 NORMAL COMPLETION
**** MODEL STATUS      1 OPTIMAL
**** OBJECTIVE VALUE                 90.0000

**** REPORT SUMMARY :        0     NONOPT
                             0 INFEASIBLE
                             0  UNBOUNDED

----     264 PARAMETER V1        =         0.0 INFEASIBILITY FLAG 1
             PARAMETER V2        =         0.0 INFEASIBILITY FLAG 2
             PARAMETER V3        =         0.0 INFEASIBILITY FLAG 3
             PARAMETER V4        =         0.0 INFEASIBILITY FLAG 4
             PARAMETER V5        =         0.0 INFEASIBILITY FLAG 5

----     267 VARIABLE  X.L     NUMBER OF AIRCRAFT OF TYPE O TO LAUNCH
                               IN CYCLE E ON A LENGTH OF M CYCLES
```

	F14.L1	F18.L1	A6.L1	A6.L2	EA6.L1	EA6.L2
1	6	8	4		1	
2	6	8	4	2		1
3	6	8		3		1
4	4	8	4			

+	E2.L1	E2.L2	S3.L1	S3.L2	ES3.L1
1		2	3		1
2				1	1
3		1	1	1	1
4	2		1		1

30 JUL 1990

CYCLE	START TIME	STOP TIME	AIRCRAFT LAUNCH/RECOVER		ESTIMATED POSIT AT END OF CYCLE		PIM POSIT AT END OF CYCLE		BEARING/DIST EP to PIM
1	1200	1345	26	1	23-07N	159-00W	23-04N	159-05W	241T/ 6NM
2	1345	1530	24	24	23-29N	158-47W	23-29N	159-06W	269T/ 18NM
3	1530	1715	23	22	23-44N	158-37W	23-53N	159-07W	288T/ 28NM
4	1715	1900	21	21	23-59N	158-23W	24-18N	159-08W	294T/ 45NM
Final	1900	1951	1	27	24-19N	158-38W	24-29N	159-08W	291T/ 30NM
Next	90212	0100			25-29N	159-59W	25-30N	160-00W	304T/ 0NM

30 JUL 1990

CYCLE 1 START TIME: 1200 STOPTIME: 1345

Scheduled Launches: 26 Scheduled Recoveries: 1

A/C TYPE	SINGLE CYCLE	DOUBLE CYCLE	TRIPLE CYCLE	YO-YO	NON ORIG	NON RET
F14	6	0	0	0	0	0
F18	8	0	0	0	0	0
A6	4	0	0	0	0	0
EA6	1	0	0	0	0	0
E2	0	2	0	0	0	0
S3	3	0	0	0	0	0
ES3	1	0	0	0	0	0
TKR	0	0	0	1	0	0
COD	0	0	0	0	0	0
SH3	0	0	0	0	0	0

30 JUL 1990

CYCLE 2 START TIME: 1345 STOPTIME: 1530

Scheduled Launches: 24 Scheduled Recoveries: 24

A/C TYPE	SINGLE CYCLE	DOUBLE CYCLE	TRIPLE CYCLE	YO-YO	NON ORIG	NON RET
F14	6	0	0	0	0	0
F18	8	0	0	0	0	0
A6	4	2	0	0	0	0
EA6	0	1	0	0	0	0
E2	0	0	0	0	0	0
S3	0	1	0	0	0	0
ES3	1	0	0	0	0	0
TKR	0	0	0	1	0	0
COD	0	0	0	0	0	0
SH3	0	0	0	0	0	0

30 JUL 1990

CYCLE 3 START TIME: 1530 STOPTIME: 1715

Scheduled Launches: 23 Scheduled Recoveries: 22

A/C TYPE	SINGLE CYCLE	DOUBLE CYCLE	TRIPLE CYCLE	YO-YO	NON ORIG	NON RET
F14	6	0	0	0	0	0
F18	8	0	0	0	0	0
A6	0	3	0	0	0	0
EA6	0	1	0	0	0	0
E2	0	1	0	0	0	0
S3	1	1	0	0	0	0
ES3	1	0	0	0	0	0
TKR	0	0	0	1	0	0
COD	0	0	0	0	0	0
SH3	0	0	0	0	0	0

30 JUL 1990

CYCLE 4 START TIME: 1715 STOPTIME: 1900

Scheduled Launches: 21 Scheduled Recoveries: 21

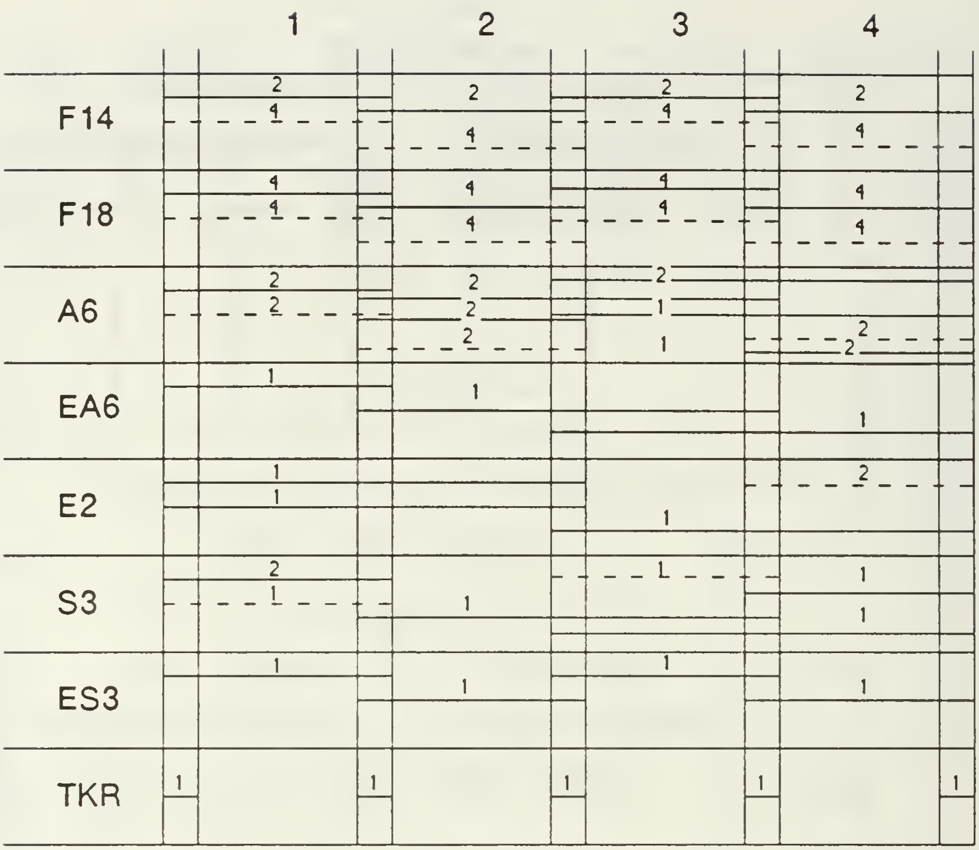
A/C TYPE	SINGLE CYCLE	DOUBLE CYCLE	TRIPLE CYCLE	YO-YO	NON ORIG	NON RET
F14	4	0	0	0	0	0
F18	8	0	0	0	0	0
A6	4	0	0	0	0	0
EA6	0	0	0	0	0	0
E2	2	0	0	0	0	0
S3	1	0	0	0	0	0
ES3	1	0	0	0	0	0
TKR	0	0	0	1	0	0
COD	0	0	0	0	0	0
SH3	0	0	0	0	0	0

30 JUL 1990

LAST RECOVERY: START TIME: 1900 STOPTIME: 1951

Scheduled Launches: 1 Scheduled Recoveries: 27

A/C TYPE	SINGLE CYCLE	DOUBLE CYCLE	TRIPLE CYCLE	YO-YO	NON ORIG	NON RET
F14	0	0	0	0	0	0
F18	0	0	0	0	0	0
A6	0	0	0	0	0	0
EA6	0	0	0	0	0	0
E2	0	0	0	0	0	0
S3	0	0	0	0	0	0
ES3	0	0	0	0	0	0
TKR	0	0	0	1	0	0
COD	0	0	0	0	0	0
SH3	0	0	0	0	0	0



Extra Sorties

Cycle	Single	Double
1	11	0
2	10	0
3	9	0
4	10	0

Using MIP on Mainframe

APPENDIX F

GENERAL ALGEBRAIC MODELING SYSTEM

```
MODEL STATISTICS    SOLVE COLP USING RMIP FROM LINE 235

               S O L V E      S U M M A R Y

     MODEL   COLA                  OBJECTIVE  Z
     TYPE    RMIP                  DIRECTION  MINIMIZE
     SOLVER  ZOOM                  FROM LINE  235

**** SOLVER STATUS     1 NORMAL COMPLETION
**** MODEL STATUS      1 OPTIMAL
**** OBJECTIVE VALUE                44.0000

**** REPORT SUMMARY :        0     NONOPT
                             0 INFEASIBLE
                             0  UNBOUNDED

----     241 PARAMETER V1        =         0.0 INFEASIBILITY FLAG 1
             PARAMETER V2        =         0.0 INFEASIBILITY FLAG 2
             PARAMETER V3        =         0.0 INFEASIBILITY FLAG 3
             PARAMETER V4        =         0.0 INFEASIBILITY FLAG 4
             PARAMETER V5        =         0.0 INFEASIBILITY FLAG 5

----     244 VARIABLE  X.L      NUMBER OF AIRCRAFT OF TYPE O TO LAUNCH
                                IN CYCLE E ON A LENGTH OF M CYCLES

         F14.L1      F18.L1      A6.L2     EA6.L2      E2.L2

1             2           4          2          1          2
2             2           4          2
3             2           4          3          1          1
4             2           4

+         S3.L1       S3.L2     ES3.L1

1             1           1          1
2                                    1
3                         1          1
4             1                      1
```

30 JUL 1990

CYCLE	START TIME	STOP TIME	AIRCRAFT LAUNCH	AIRCRAFT RECOVER	ESTIMATED POSIT AT END OF CYCLE		PIM POSIT AT END OF CYCLE		BEARING/DIST EP to PIM
1	1200	1345	15	1	23-08N	159-02W	23-04N	159-05W	221T/ 5NM
2	1345	1530	10	9	23-35N	158-57W	23-29N	159-06W	232T/ 10NM
3	1530	1715	14	14	23-55N	158-52W	23-53N	159-07W	260T/ 14NM
4	1715	1900	9	10	24-17N	158-49W	24-18N	159-08W	272T/ 18NM
Final	1900	1931	1	15	24-29N	158-55W	24-25N	159-08W	249T/ 13NM
Next	90212	0100			25-30N	160-00W	25-30N	160-00W	000T/ 0NM

30 JUL 1990

CYCLE 1 START TIME: 1200 STOPTIME: 1345

Scheduled Launches: 15 Scheduled Recoveries: 1

A/C TYPE	SINGLE CYCLE	DOUBLE CYCLE	TRIPLE CYCLE	YO-YO	NON ORIG	NON RET
F14	2	0	0	0	0	0
F18	4	0	0	0	0	0
A6	0	2	0	0	0	0
EA6	0	1	0	0	0	0
E2	0	2	0	0	0	0
S3	1	1	0	0	0	0
ES3	1	0	0	0	0	0
TKR	0	0	0	1	0	0
COD	0	0	0	0	0	0
SH3	0	0	0	0	0	0

30 JUL 1990

CYCLE 2 START TIME: 1345 STOPTIME: 1530

Scheduled Launches: 10 Scheduled Recoveries: 9

A/C TYPE	SINGLE CYCLE	DOUBLE CYCLE	TRIPLE CYCLE	YO-YO	NON ORIG	NON RET
F14	2	0	0	0	0	0
F18	4	0	0	0	0	0
A6	0	2	0	0	0	0
EA6	0	0	0	0	0	0
E2	0	0	0	0	0	0
S3	0	0	0	0	0	0
ES3	1	0	0	0	0	0
TKR	0	0	0	1	0	0
COD	0	0	0	0	0	0
SH3	0	0	0	0	0	0

30 JUL 1990

CYCLE 3 START TIME: 1530 STOPTIME: 1715

Scheduled Launches: 14 Scheduled Recoveries: 14

A/C TYPE	SINGLE CYCLE	DOUBLE CYCLE	TRIPLE CYCLE	YO-YO	NON ORIG	NON RET
F14	2	0	0	0	0	0
F18	4	0	0	0	0	0
A6	0	3	0	0	0	0
EA6	0	1	0	0	0	0
E2	0	1	0	0	0	0
S3	0	1	0	0	0	0
ES3	1	0	0	0	0	0
TKR	0	0	0	1	0	0
COD	0	0	0	0	0	0
SH3	0	0	0	0	0	0

30 JUL 1990

CYCLE 4 START TIME: 1715 STOPTIME: 1900

Scheduled Launches: 9 Scheduled Recoveries: 10

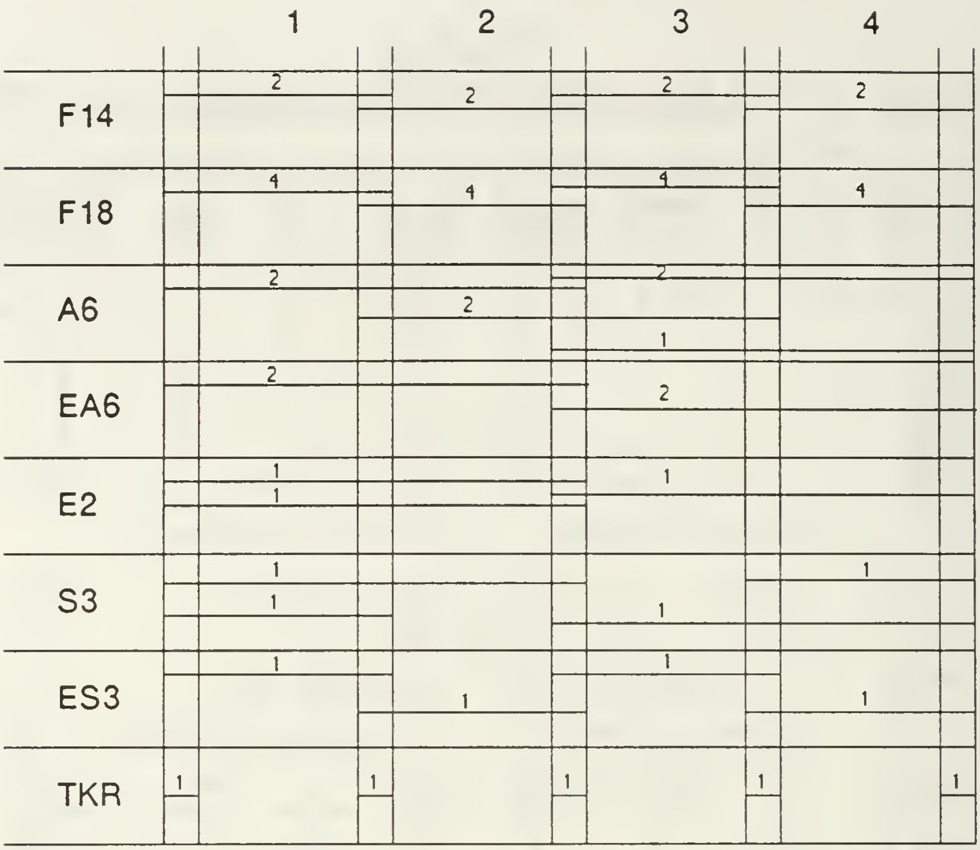
A/C TYPE	SINGLE CYCLE	DOUBLE CYCLE	TRIPLE CYCLE	YO-YO	NON ORIG	NON RET
F14	2	0	0	0	0	0
F18	4	0	0	0	0	0
A6	0	0	0	0	0	0
EA6	0	0	0	0	0	0
E2	0	0	0	0	0	0
S3	1	0	0	0	0	0
ES3	1	0	0	0	0	0
TKR	0	0	0	1	0	0
COD	0	0	0	0	0	0
SH3	0	0	0	0	0	0

30 JUL 1990

LAST RECOVERY: START TIME: 1900 STOPTIME: 1931

Scheduled Launches: 1 Scheduled Recoveries: 15

A/C TYPE	SINGLE CYCLE	DOUBLE CYCLE	TRIPLE CYCLE	YO-YO	NON ORIG	NON RET
F14	0	0	0	0	0	0
F18	0	0	0	0	0	0
A6	0	0	0	0	0	0
EA6	0	0	0	0	0	0
E2	0	0	0	0	0	0
S3	0	0	0	0	0	0
ES3	0	0	0	0	0	0
TKR	0	0	0	1	0	0
COD	0	0	0	0	0	0
SH3	0	0	0	0	0	0


1
2
3
4
F14
F18
A6
EA6
E2
S3
ES3
TKR

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